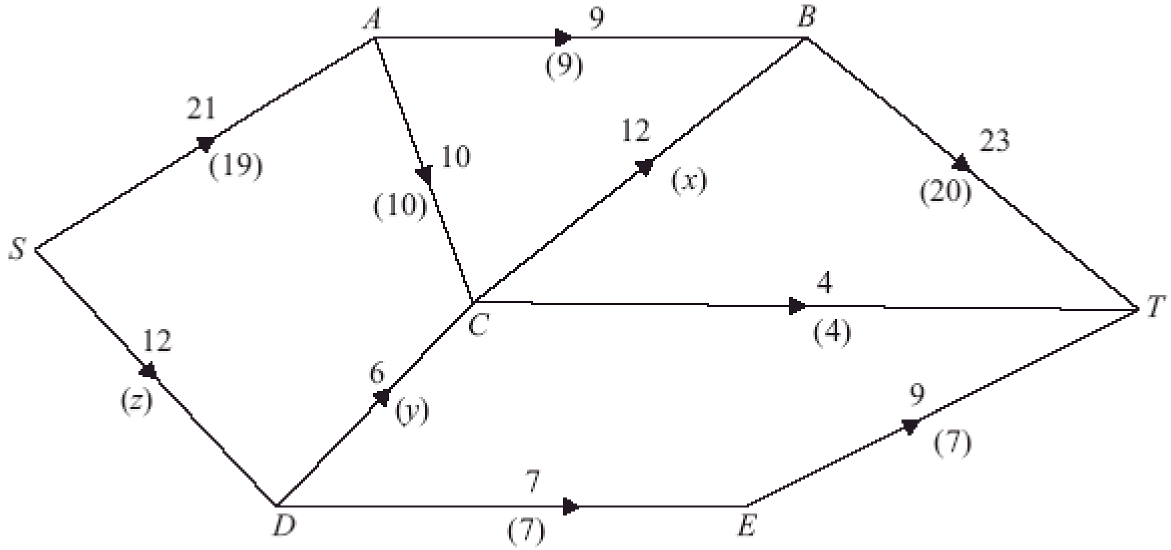
1.



The figure above shows a directed, capacitated network where the number on each arc is its capacity. A possible flow is shown from S to T and the value in brackets on each arc is the flow in that arc.

(a) Find the values of x, y, and z.

(3)

(b) Find, by inspection, the maximal flow from S to T and verify that it is maximal.

(2)
(Total 5 marks)

2.

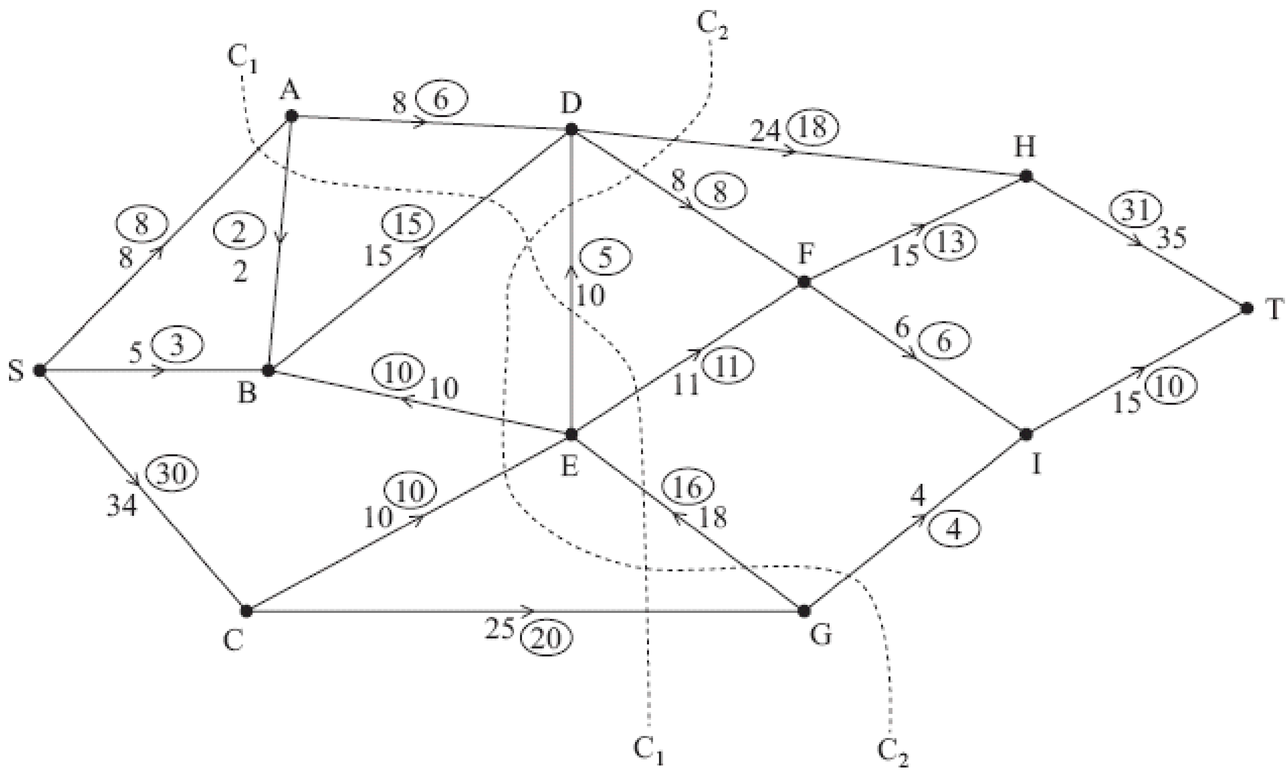


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow. **(1)**

(b) State the capacities of cuts C_1 and C_2 **(2)**

(c) By entering values along DH, FH, FI and IT, complete the labelling procedure on Figure 2.

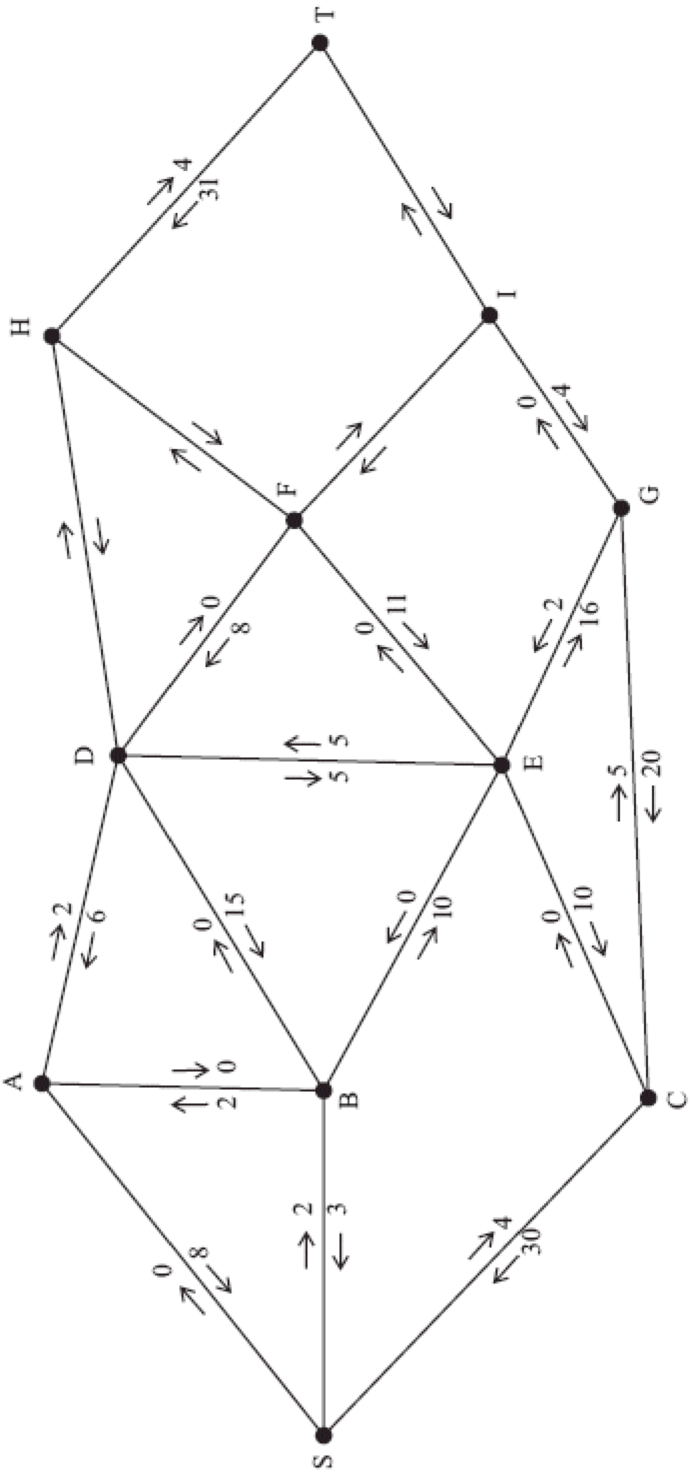


Figure 2

(2)

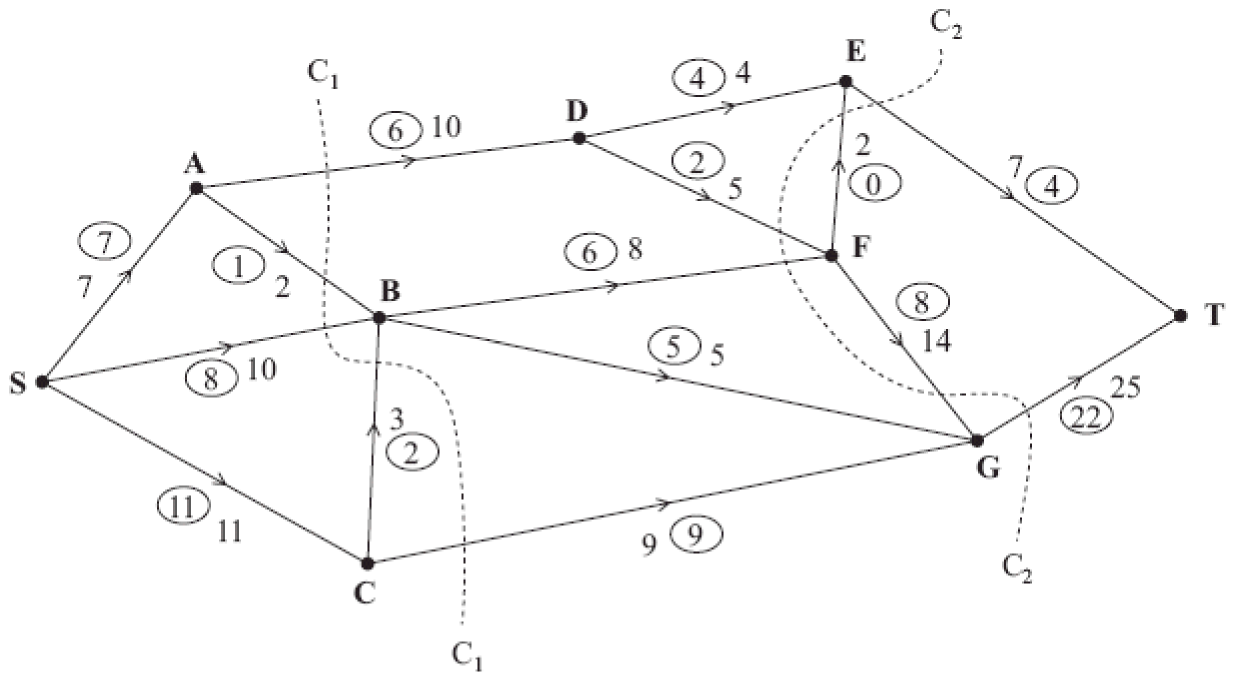
(d) Using Figure 2, increase the flow by a further 4 units. You **must** list each flow-augmenting route you use, together with its flow.

(3)

(e) Prove that the flow is now maximal.

(2)
(Total 10 marks)

3.



The diagram above shows a capacitated network. The capacity of each arc is shown on the arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown in the diagram.

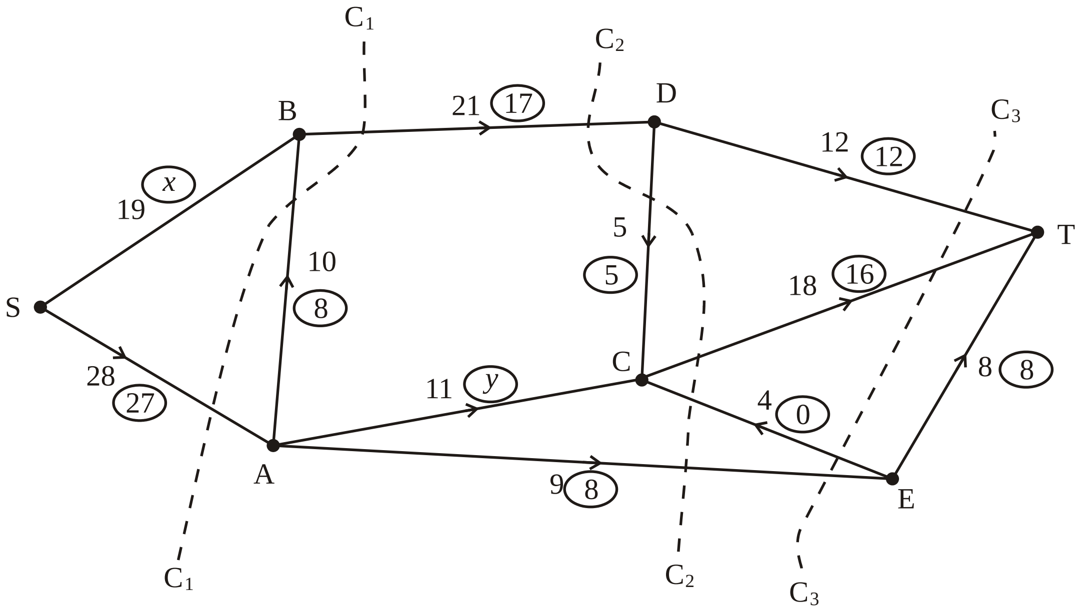
(a) Find the capacity of each of the two cuts.

(2)

(b) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

(3)
(Total 5 marks)

4.



The diagram above shows a capacitated, directed network of pipes. The number on each arc represents the capacity of that pipe. The numbers in circles represent a feasible flow.

(a) State the values of *x* and *y*.

(2)

(b) List the saturated arcs.

(2)

(c) State the value of the feasible flow.

(1)

(d) State the capacities of the cuts C_1, C_2, and C_3.

(3)

(e) By inspection, find a flow-augmenting route to increase the flow by one unit. You must state your route.

(1)

(f) Prove that the new flow is maximal.

(2)
(Total 11 marks)

5. (a) Define the term ‘cut’ as it applies to a directed network.

(2)

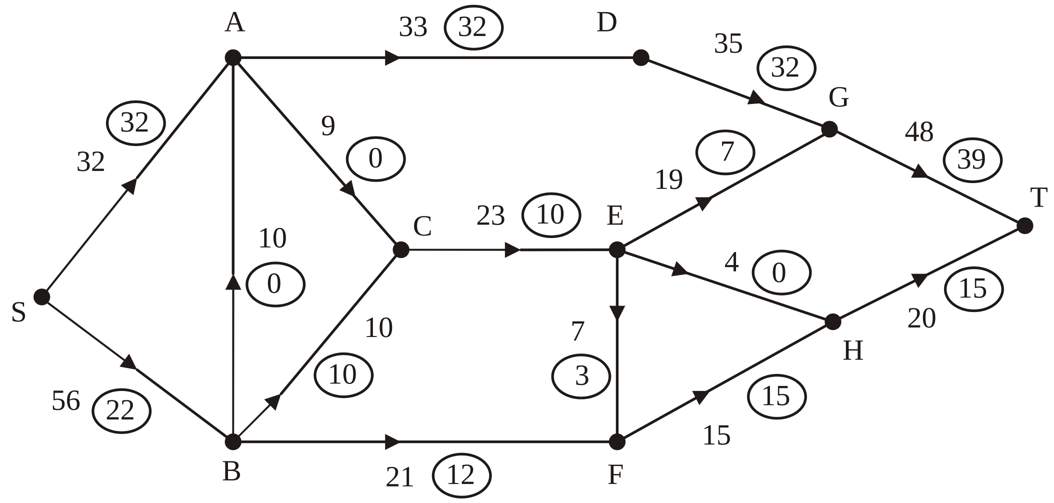


Diagram 1

Diagram 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(b) Complete the labelling procedure on Diagram 2 below by entering values along CE, EG, HT and GT.

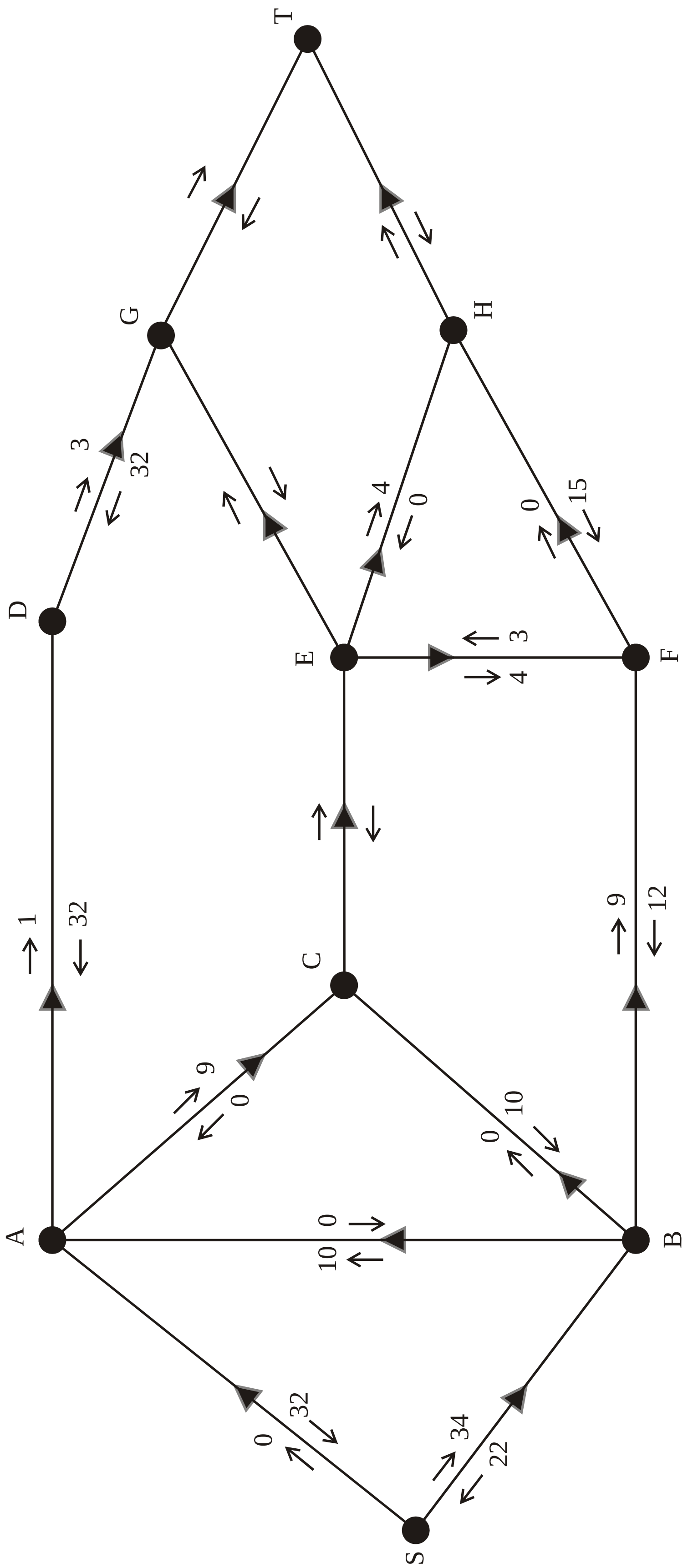


Diagram 2

(2)

(c) Find the maximum flow through the network. You must list each flow- augmenting route you use together with its flow.

(4)

(d) Show a maximal flow pattern on Diagram 3.

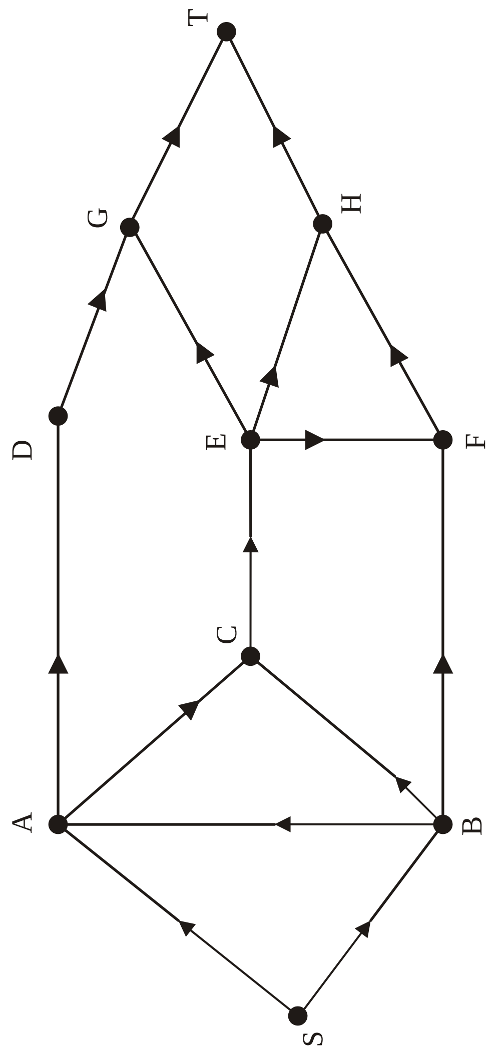


Diagram 3

(2)

(e) State the value of the maximum flow through the network.

(1)

(f) Prove that your flow is maximal.

(2)
(Total 13 marks)

6.

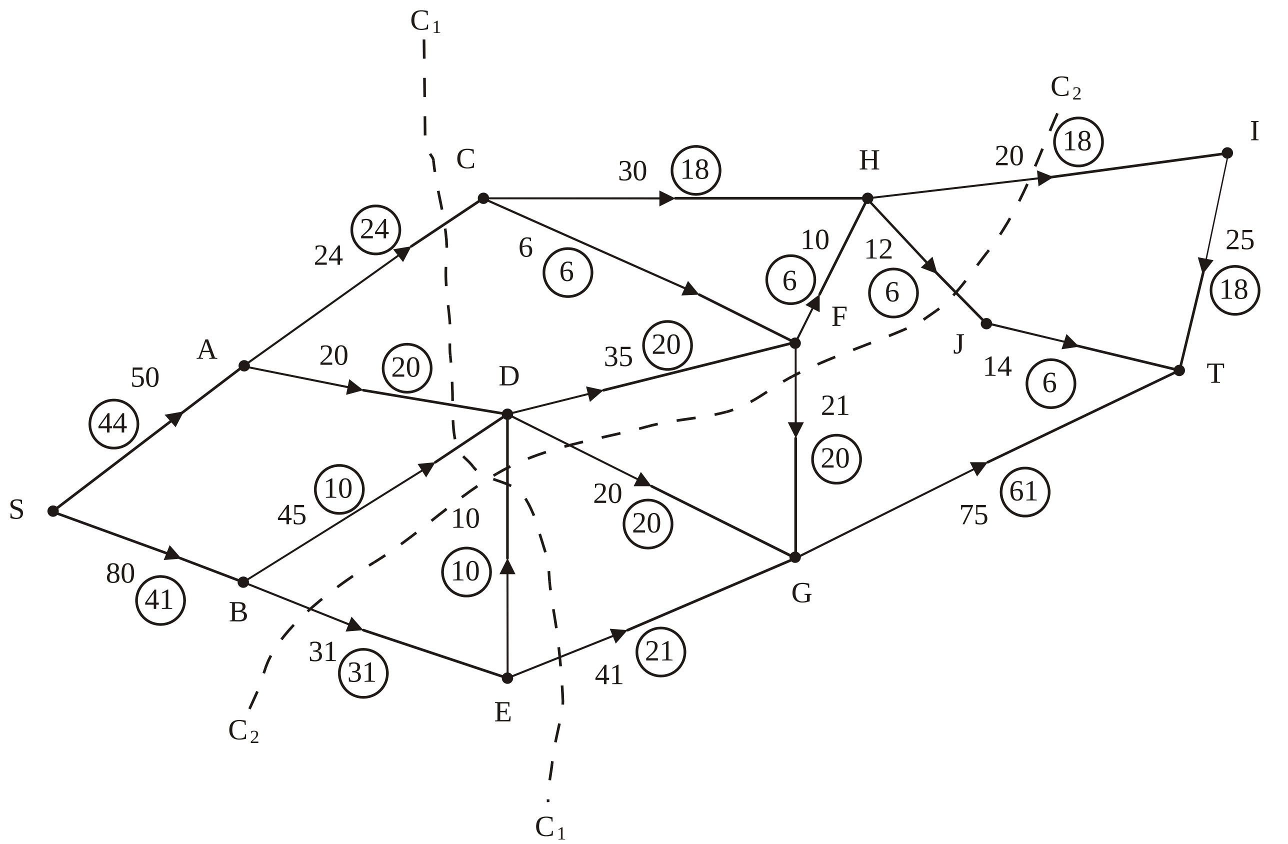


Figure 1

Figure 1 shows a capacitated, directed network. The number on each arc represents the capacity of that arc. The numbers in circles represent an initial flow.

(a) State the value of the initial flow.

(1)

(b) State the capacities of cuts C_1 and C_2.

(2)

Figure 2 shows the labelling procedure applied to the above network.

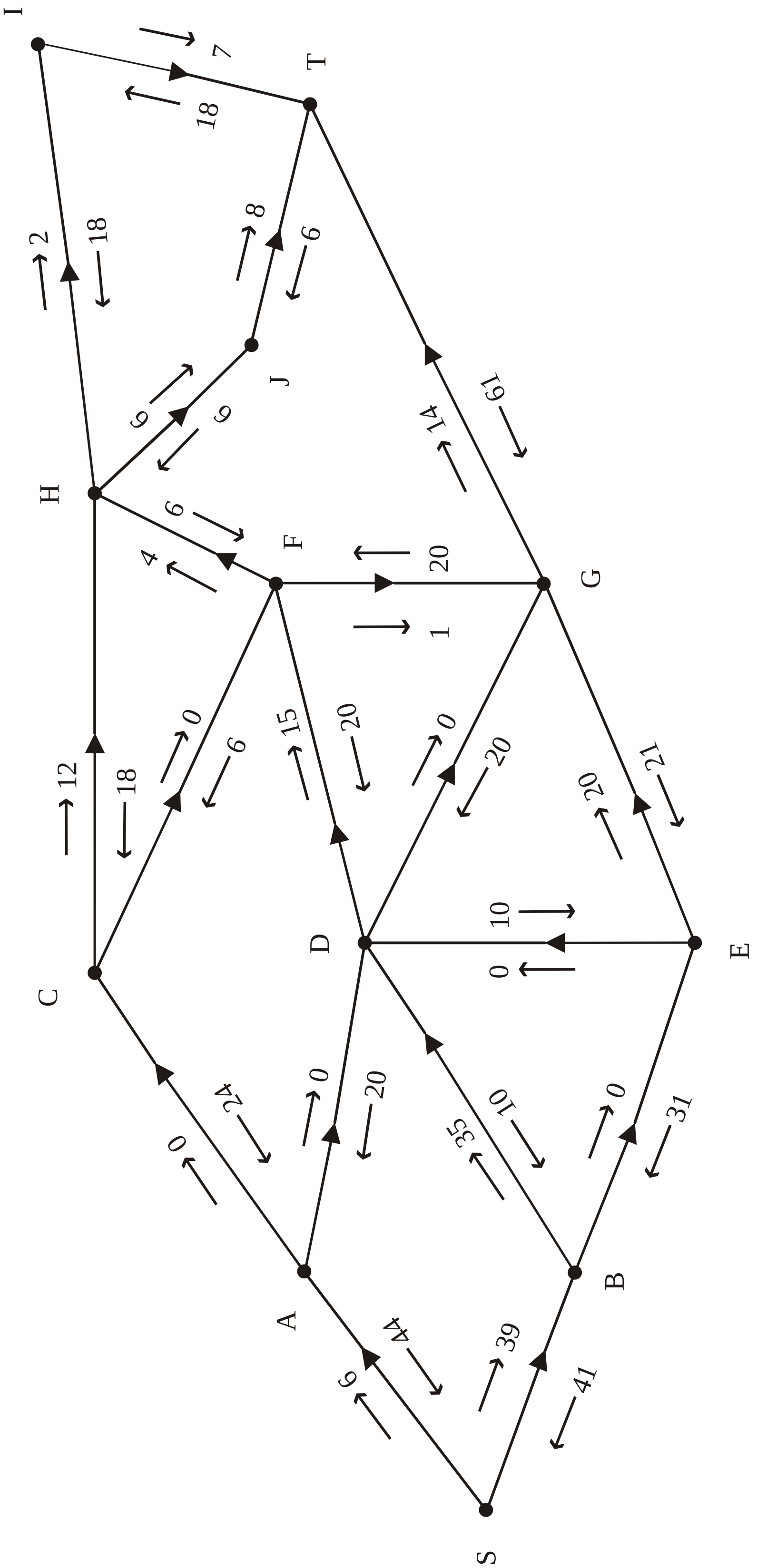


Figure 2

(c) Using Figure 2, increase the flow by a further 19 units. You **must** list each flow-augmenting path you use, together with its flow.

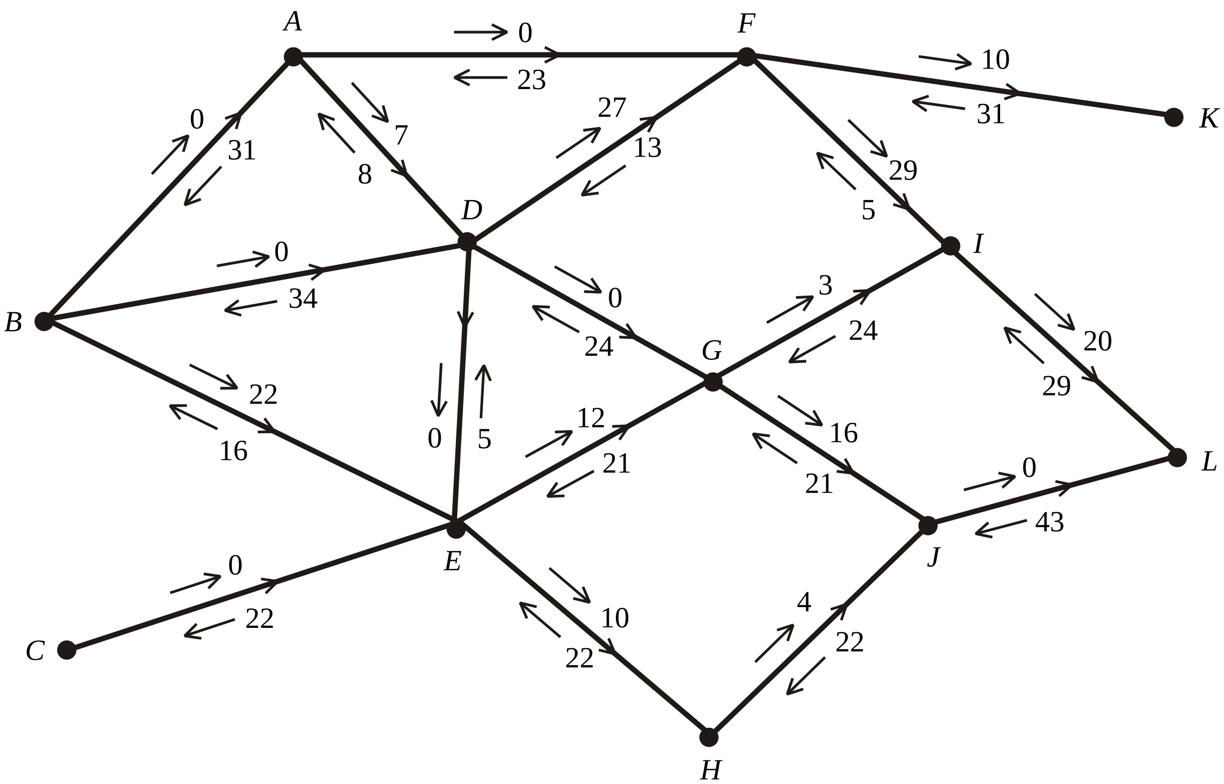
(5)

(d) Prove that the flow is now maximal.

(2)
(Total 10 marks)

7.

Figure 1



In solving a network flow problem using the labelling procedure, the diagram in Figure 1 was created.
The arrow on each arc indicates the direction of the flow along that arc.
The arrows above and below each arc show the direction and value of the flow as indicated by the labelling procedure.

(a) Add a supersource S, a supersink T and appropriate arcs to the diagram above, and complete the labelling procedure for these arcs.

(3)

(b) Write down the value of the initial flow shown in Figure 1.

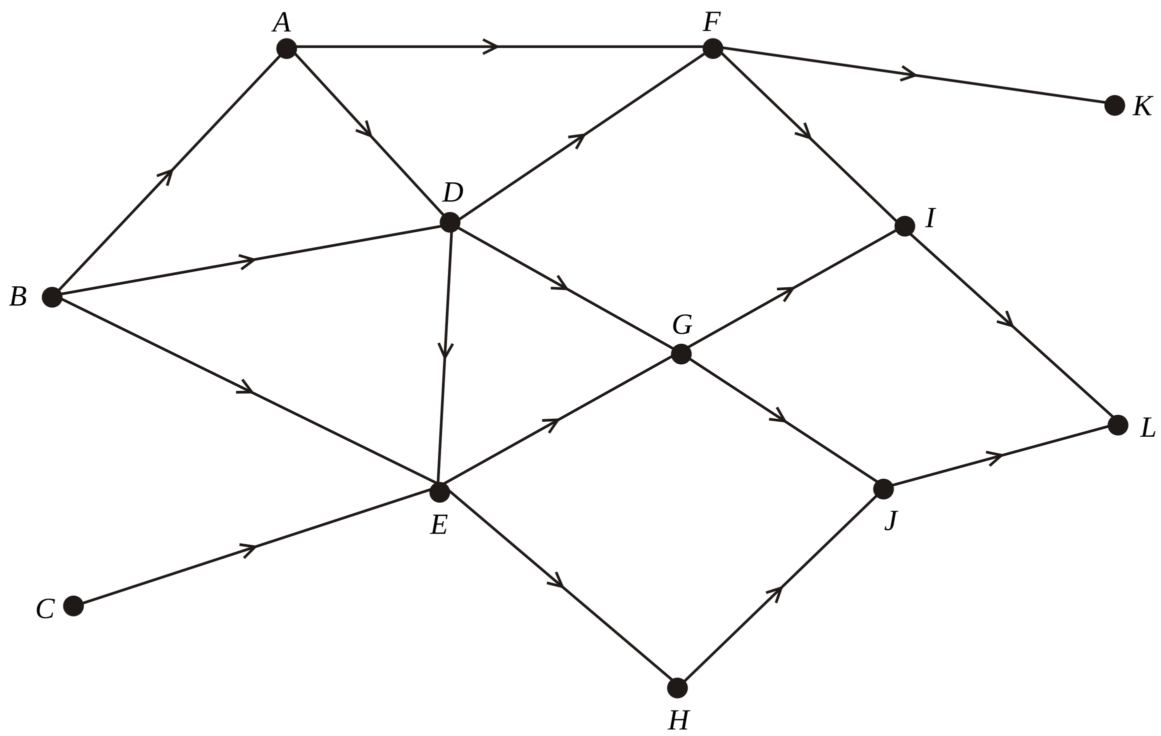
(1)

(c) Use Figure 2 below, the initial flow and the labelling procedure to find the maximal flow of 124 through this network. List each flow-augmenting path you use, together with its flow.

(5)

(d) Show your flow on the diagram below and state its value.

Figure 2

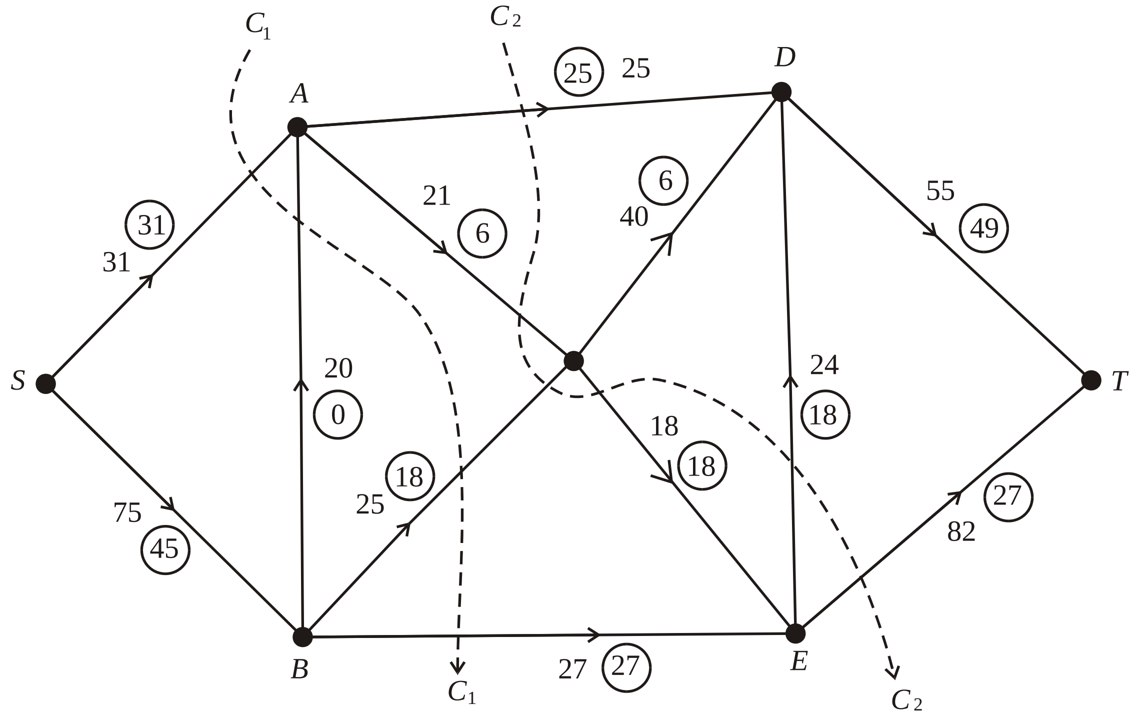


(3)

(e) Prove that your flow is maximal.

(2)
(Total 14 marks)

8.



The figure above shows a capacitated, directed network. The capacity of each arc is shown on each arc. The numbers in circles represent an initial flow from S to T.

Two cuts C_1 and C_2 are shown on the figure.

(a) Write down the capacity of each of the two cuts and the value of the initial flow.

....

....

....

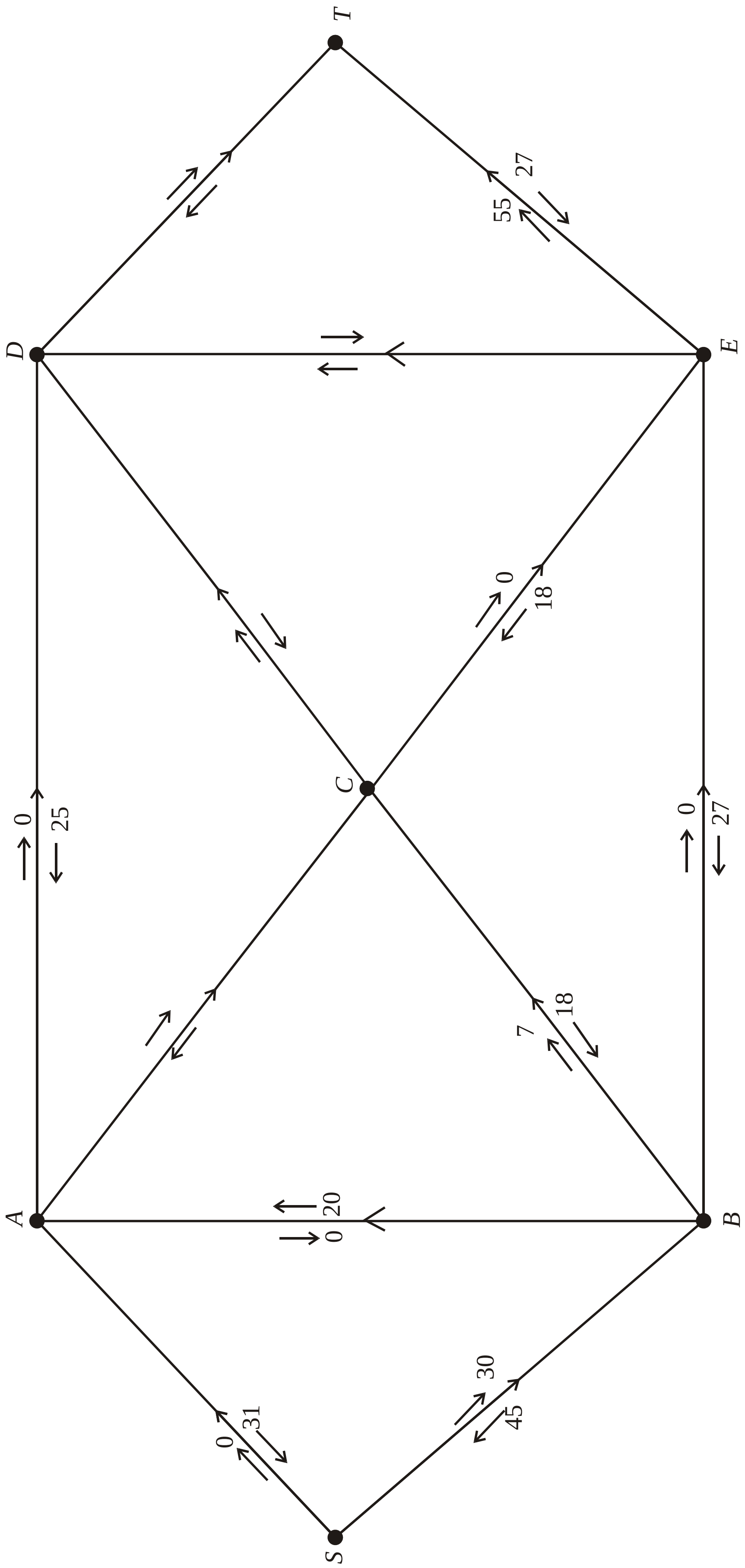
Value of C_1 ....

Value of C_2..

Value of flow......................................

(3)

(b) Complete the initialisation of the labelling procedure on the diagram below by entering values along arcs AC, CD, DE and DT.



(2)

(c) Hence use the labelling procedure to find a maximal flow through the network. You must list each flow-augmenting path you use, together with its flow.

....

....

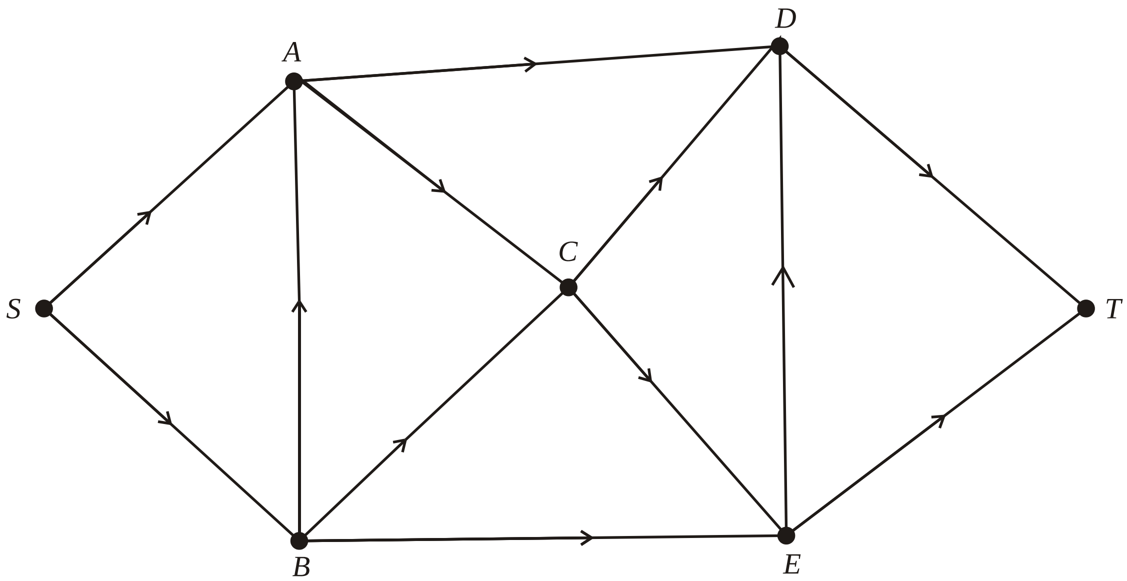
....

....

Maximal flow.......................................

(5)

(d) Show your maximal flow pattern on the diagram below.



(2)

(e) Prove that your flow is maximal.

(2)
(Total 14 marks)

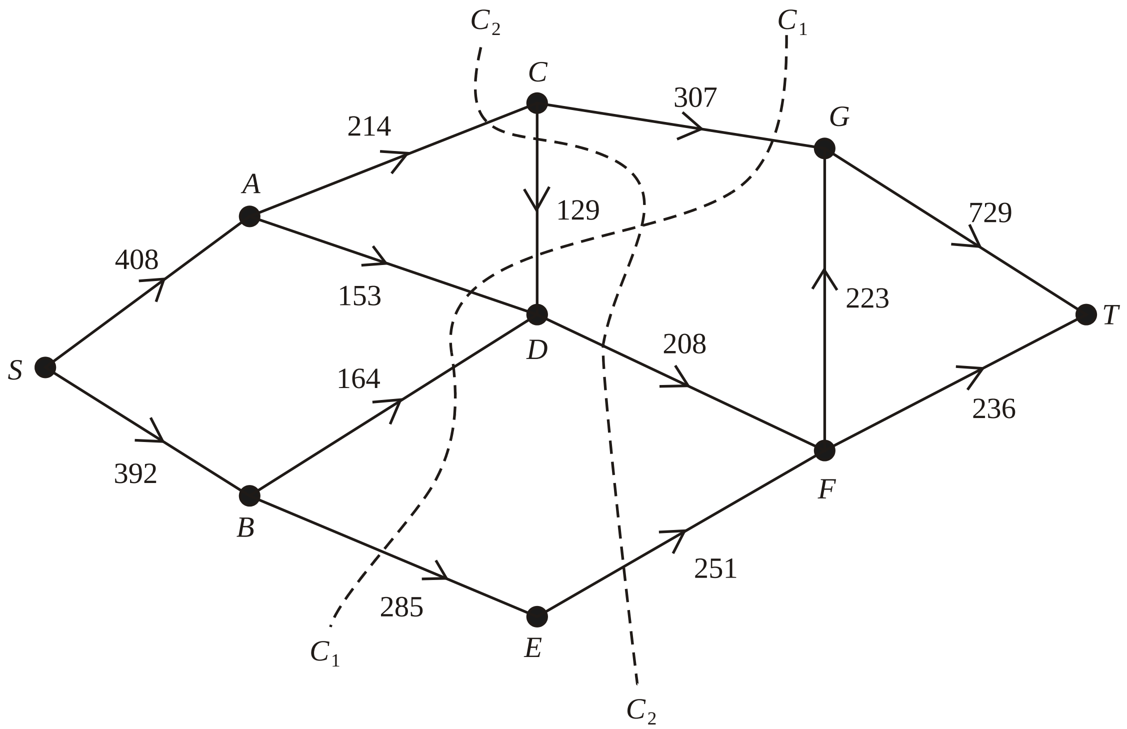
9. (a) Define the terms

(i) cut,

(ii) minimum cut,

as applied to a directed network flow.

(2)



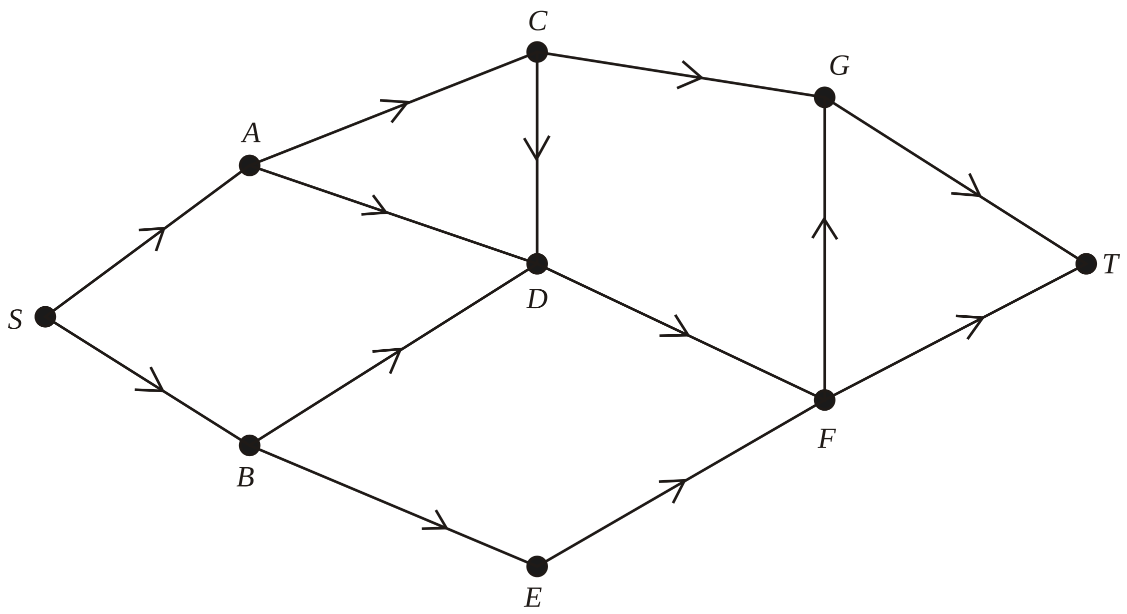
The figure above shows a capacitated directed network and two cuts C_1 and C_2. The number on each arc is its capacity.

(b) State the values of the cuts C_1 and C_2.

(3)

Given that one of these two cuts is a minimum cut,

(c) find a maximum flow pattern by inspection, and show it on the diagram below.



(3)

(d) Find a second minimum cut for this network.

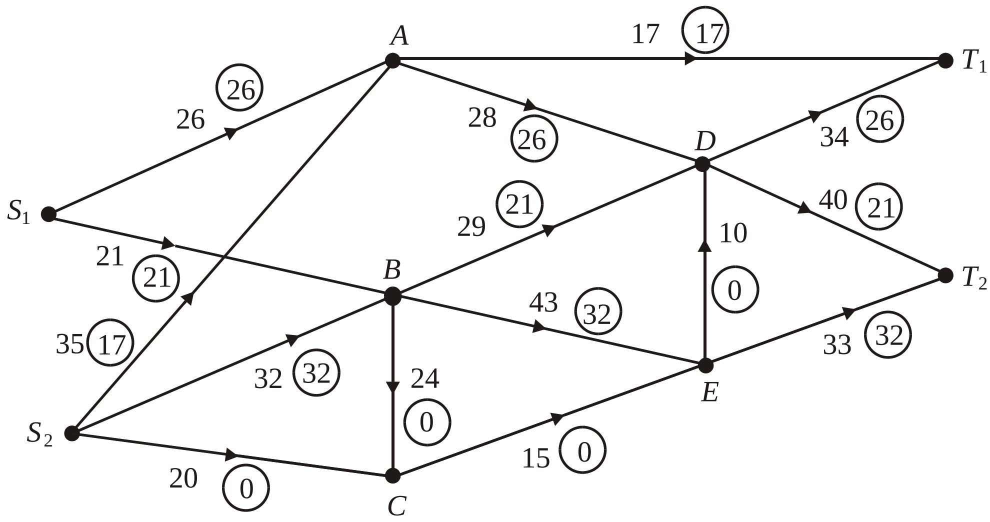
(1)

In order to increase the flow through the network it is decided to add an arc of capacity 100 joining *D* **either** to *E* **or** to *G*.

(e) State, with a reason, which of these arcs should be added, and the value of the increased flow.

(2)
(Total 11 marks)

10.



This figure shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow through the network. **Take this as the initial flow.**

(a) On Diagram 1 **and** Diagram 2 in the answer book, add a supersource *S* and a supersink *T*. On Diagram 1 show the minimum capacities of the arcs you have added.

(2)

Diagram 2 in the answer book shows the first stage of the labelling procedure for the given initial flow.

(b) Complete the initial labelling procedure in Diagram 2.

(2)

(c) Find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow, and state the maximal flow.

(6)

(d) Show a maximal flow pattern on Diagram 3.

(2)

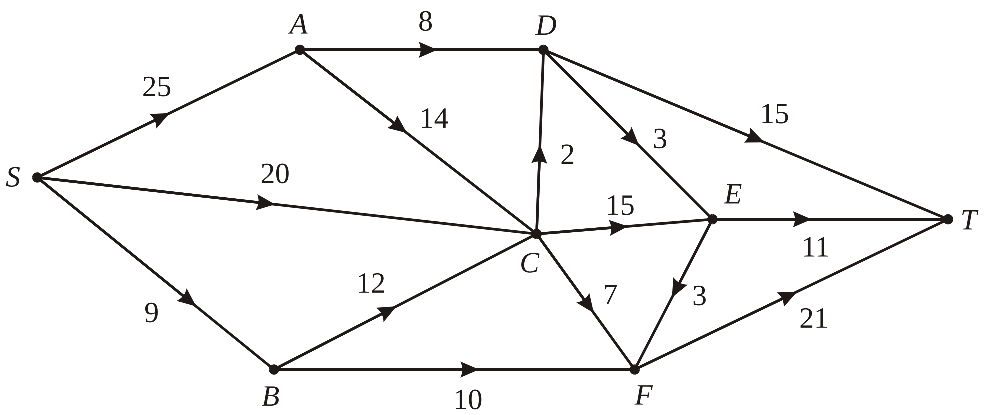
(e) Prove that your flow is maximal.

(2)

(f) Describe briefly a situation for which this network could be a suitable model.

(2)
(Total 16 marks)

11.



This diagram shows a capacitated directed network. The number on each arc is its capacity.

(a) State the maximum flow along

(i) *SADT*,

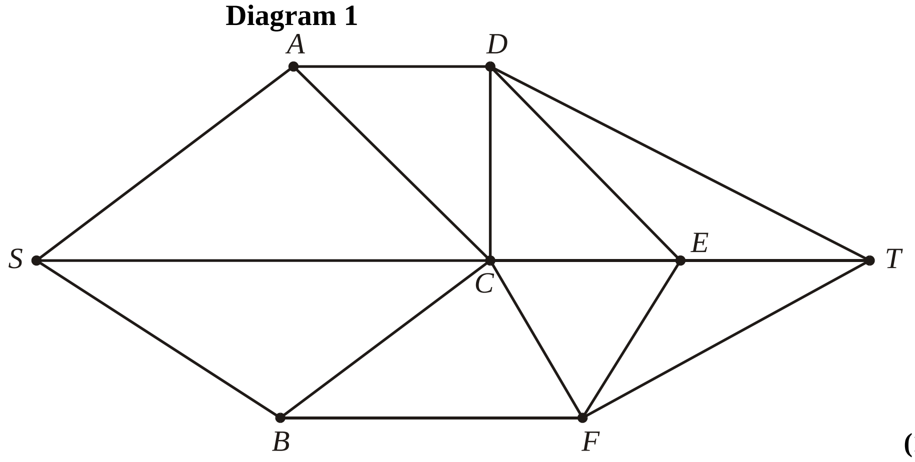
(ii) *SCET*,

(iii) *SBFT*.

(2)

(b) Show these maximum flows on Diagram 1 below.

Diagram 1

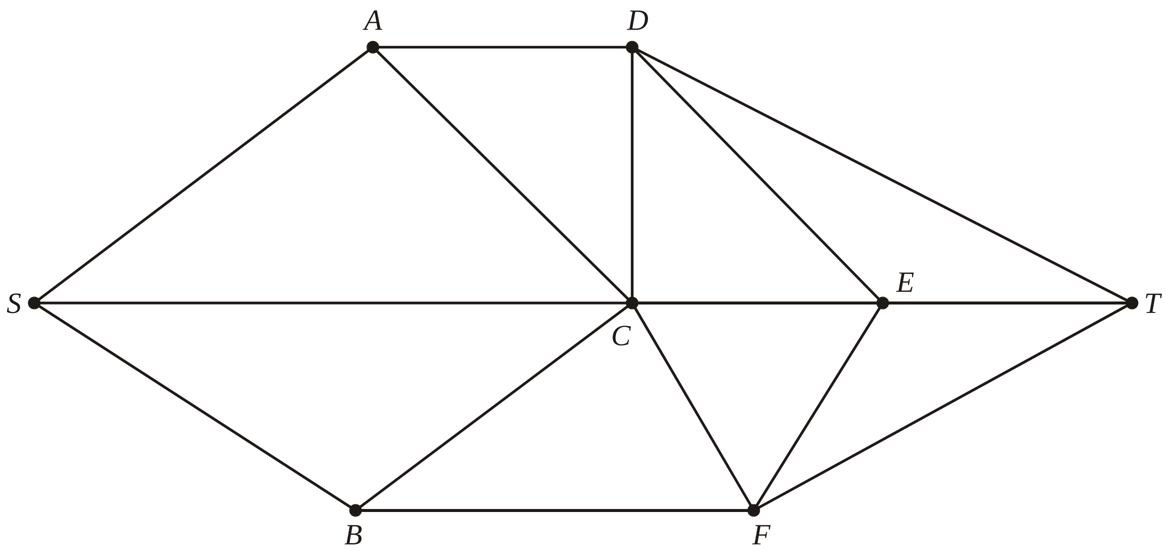


(1)

Take your answer to part (*b*) as the initial flow pattern.

(c) (i) Use the labelling procedure to find a maximum flow from S to T. Your working should be shown on Diagram 2 below. List each flow-augmenting route you use, together with its flow.

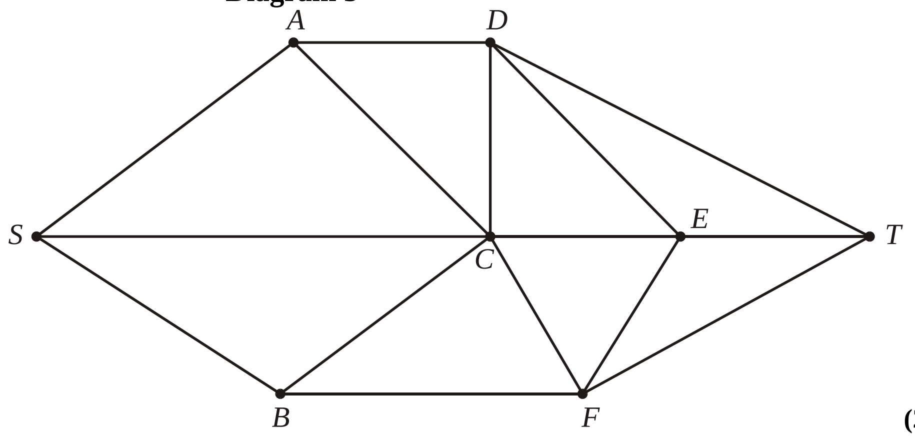
Diagram 2



(5)

(ii) Draw your final flow pattern on Diagram 3 below.

Diagram 3



(2)

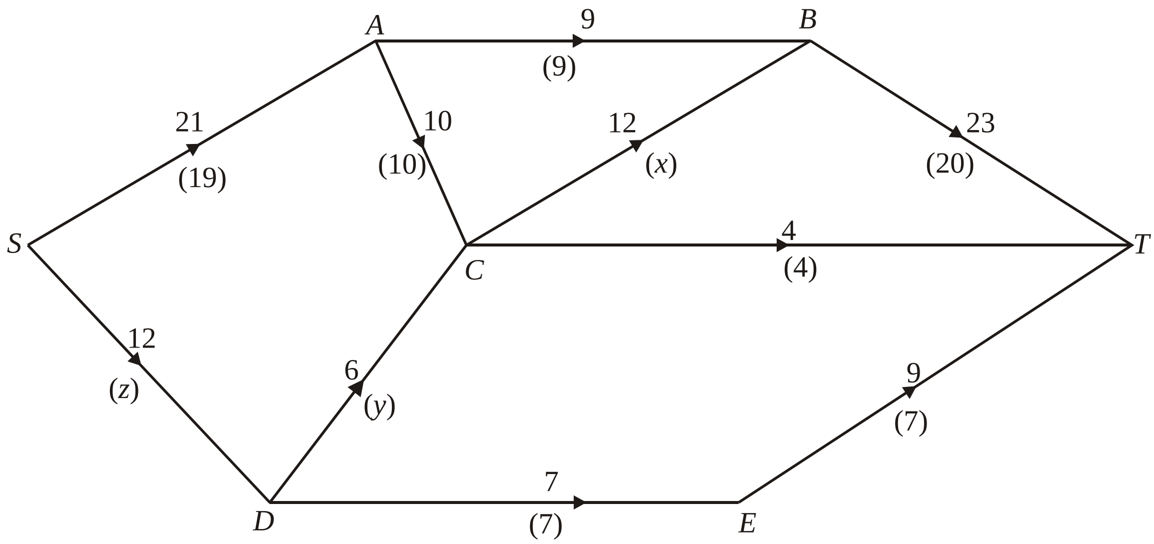
(iii) Prove that your flow is maximal.

(3)

(d) Give an example of a practical situation that could have been modelled by the original network.

(1)
(Total 14 marks)

12.



The diagram above shows a directed, capacitated network where the number on each arc is its capacity. A possible flow is shown from S to T and the value in brackets on each arc is the flow in that arc.

(a) Find the values of x, y and z.

(3)

(b) Find, by inspection, the maximal flow from S to T and verify that it is maximal.

(2)
(Total 5 marks)

13.

Figure 1

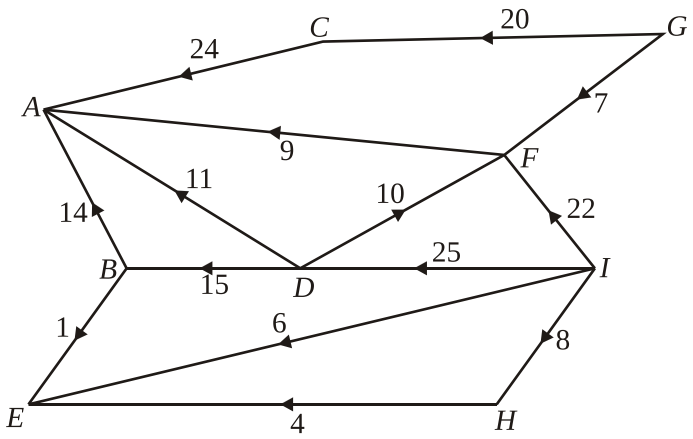


Figure 1 shows a capacitated directed network. The number on each arc is its capacity.

Figure 2

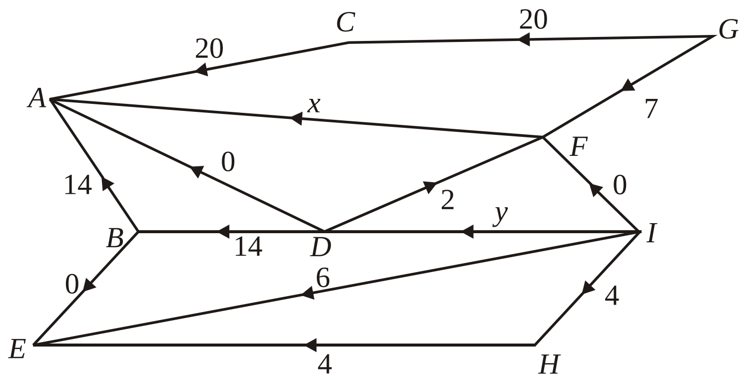


Figure 2 shows a feasible initial flow through the same network.

(a) Write down the values of the flow *x* and the flow *y*.

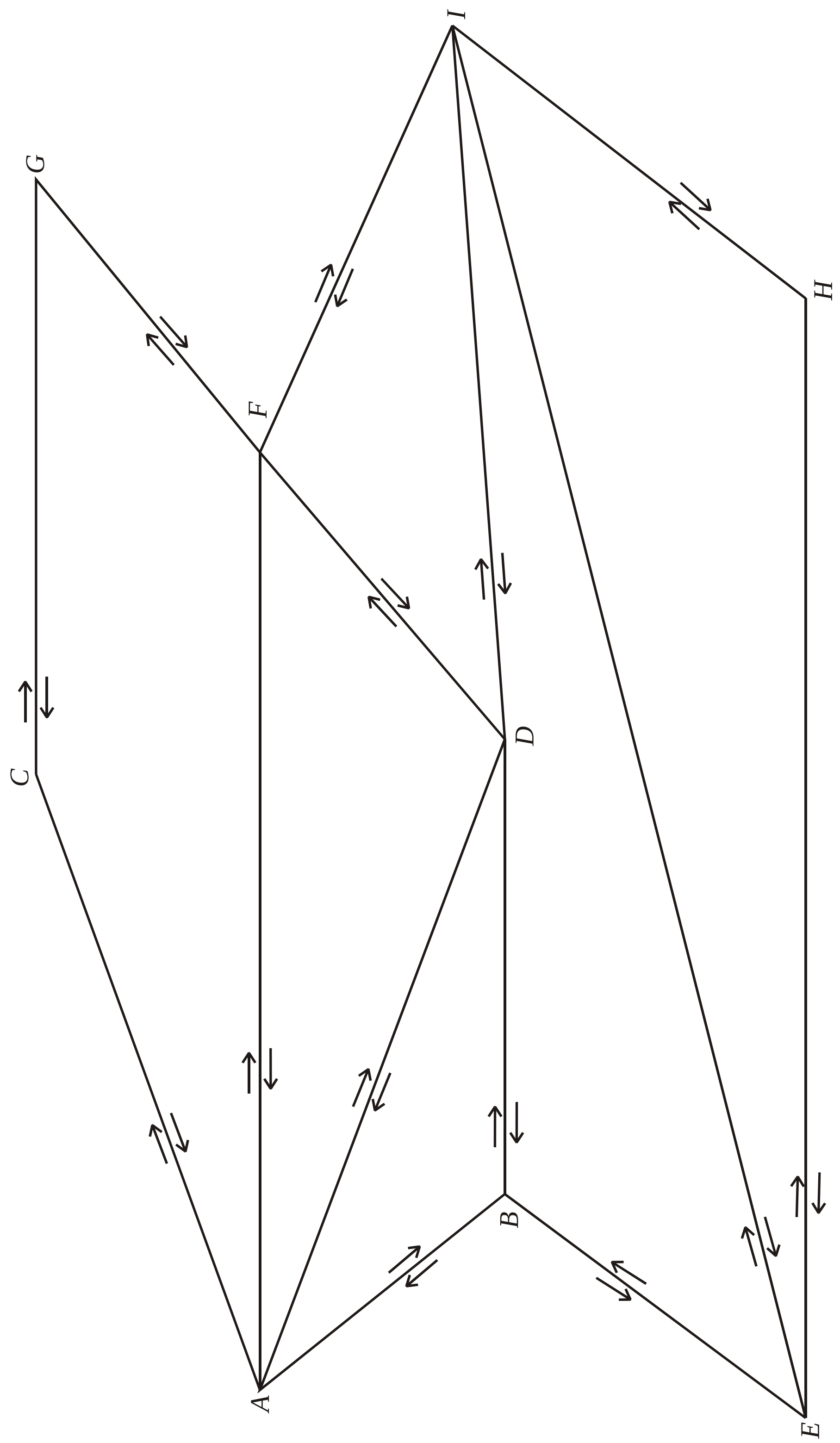
(2)

(b) Obtain the value of the initial flow through the network, and explain how you know it is not maximal.

(2)

(c) Use this initial flow and the labelling procedure on Diagram 1 below to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow.

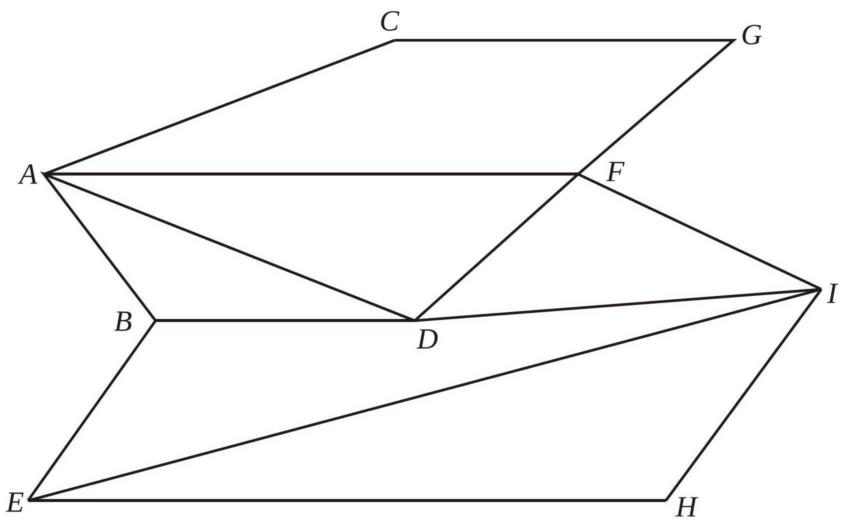
Diagram 1



(5)

(d) Show your maximal flow pattern on Diagram 2.

Diagram 2

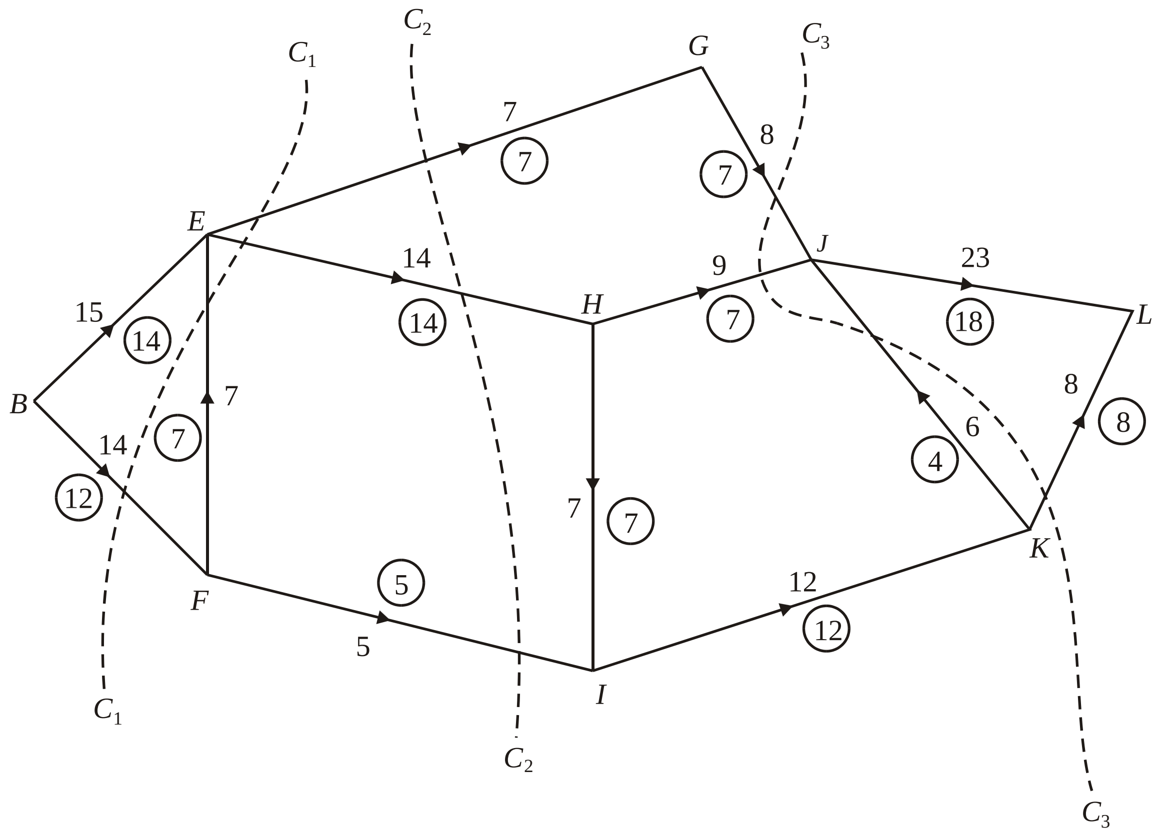


(2)

(e) Prove that your flow is maximal.

(2)
(Total 13 marks)

14.



The diagram above shows a network of roads represented by arcs. The capacity of the road represented by that arc is shown on each arc. The numbers in circles represent a possible flow of 26 from *B* to *L*.

Three cuts C_1, C_2 and C_3 are shown on The diagram above.

(a) Find the capacity of each of the three cuts.

(3)

(b) Verify that the flow of 26 is maximal.

(1)

The government aims to maximise the possible flow from *B* to *L* by using one of two options.

Option 1: Build a new road from *E* to *J* with capacity 5.

or Option 2: Build a new road from *F* to *H* with capacity 3.

(c) By considering **both** options, explain which one meets the government's aim

(3)

15.

Figure 1

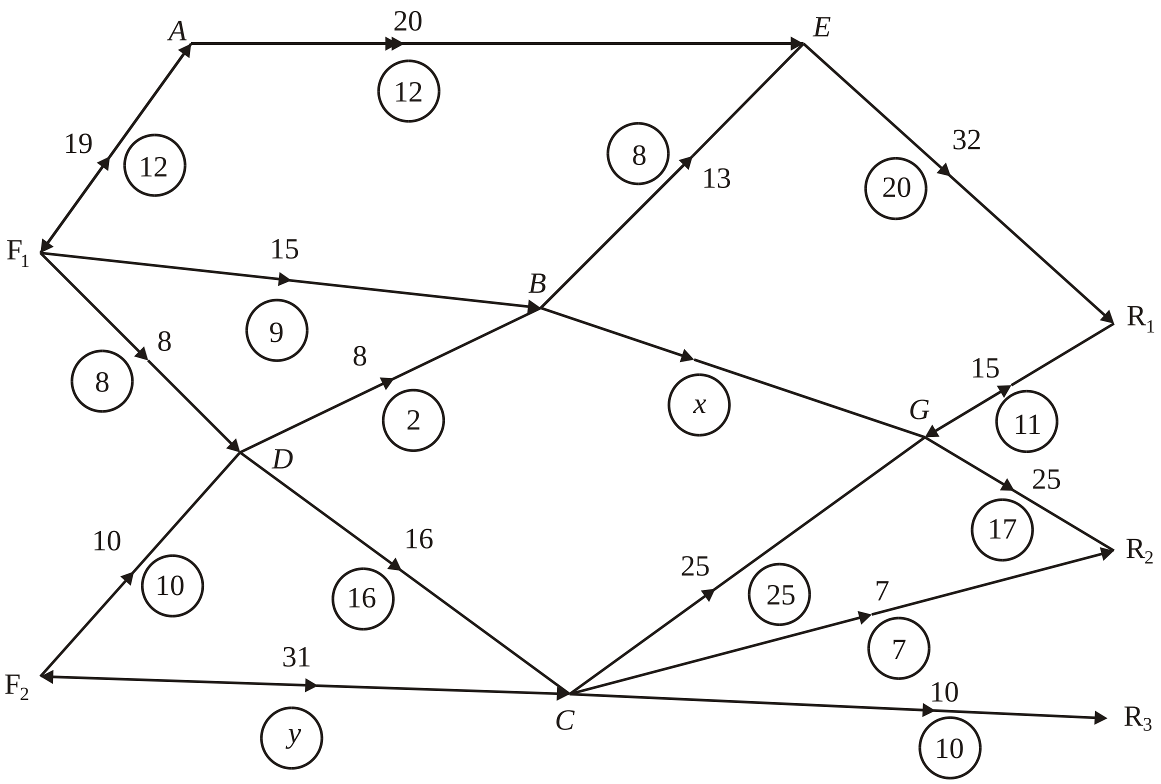


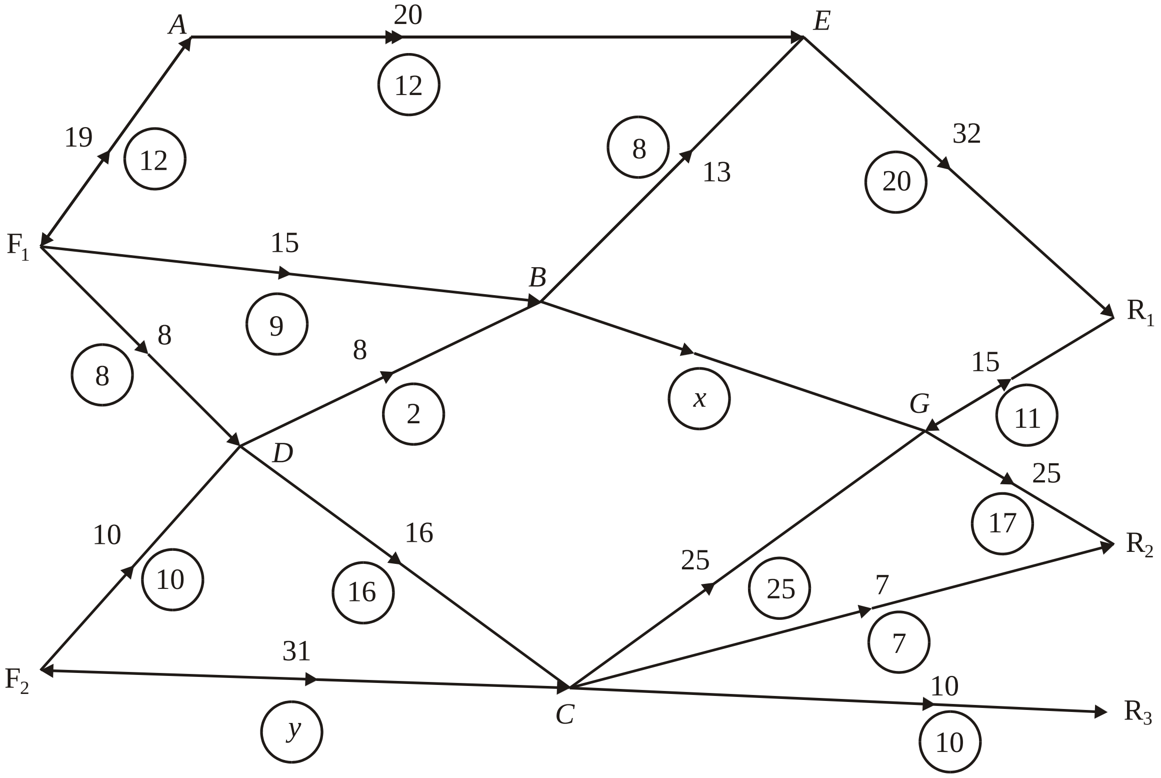
Figure 1 shows a capacitated, directed network of pipes flowing from two oil fields F_1 and F_2 to three refineries R_1, R_2 and R_3. The number on each arc represents the capacity of the pipe and the numbers in the circles represent a possible flow of 65.

(a) Find the value of x and the value of y.

(2)

(b) On Diagram 1 below, add a supersource and a supersink, and arcs showing their minimum capacities.

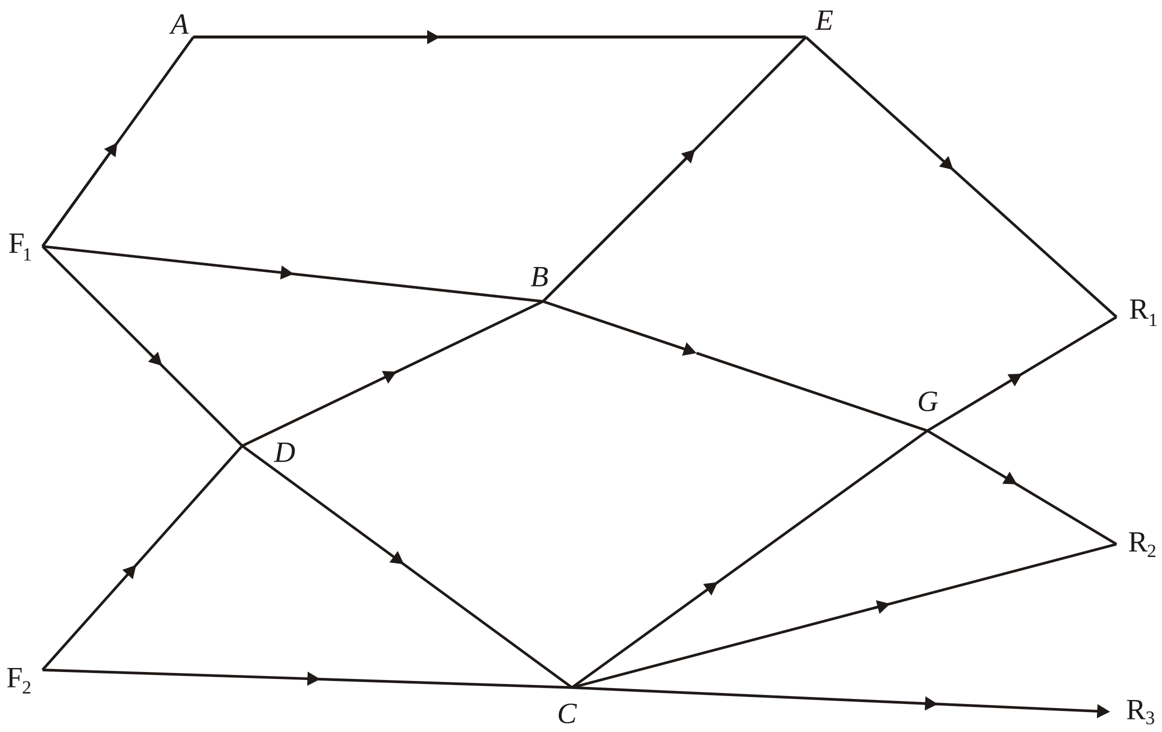
Diagram 1



(3)

(c) Taking the given flow of 65 as the initial flow pattern, use the labelling procedure on Diagram 2 to find the maximum flow. State clearly your flow augmenting routes.

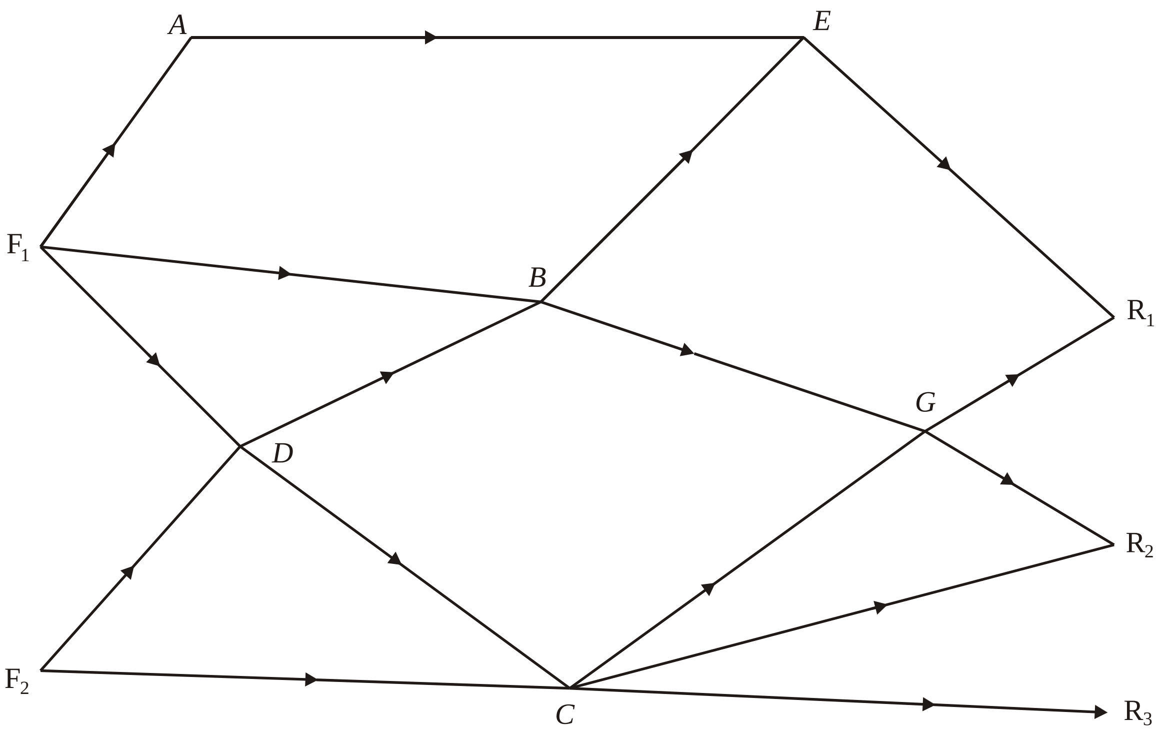
Diagram 2



(7)

(d) Show the maximum flow on Diagram 3 and write down its value.

Diagram 3



(2)

(e) Verify that this is the maximum flow by finding a cut equal to the flow.

(2)
(Total 16 marks)

16.

Figure 1

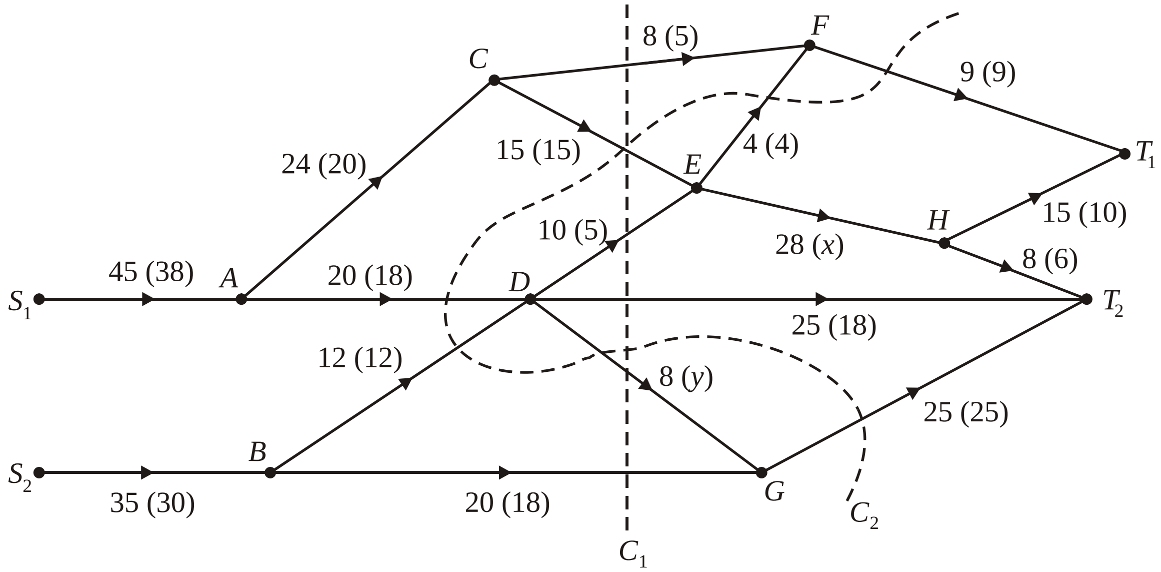
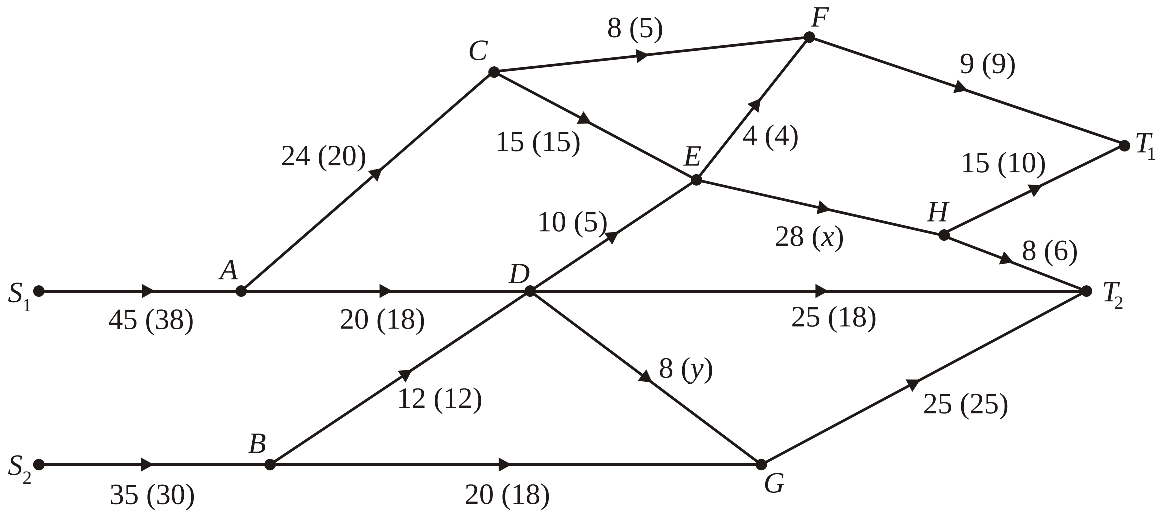


Figure 1 shows a capacitated, directed network. The unbracketed number on each arc indicates the capacity of that arc, and the numbers in brackets show a feasible flow of value 68 through the network.

(a) Add a supersource and a supersink, and arcs of appropriate capacity, to Diagram 1 below.

Diagram 1



(2)

(b) Find the values of x and y, explaining your method briefly.

(2)

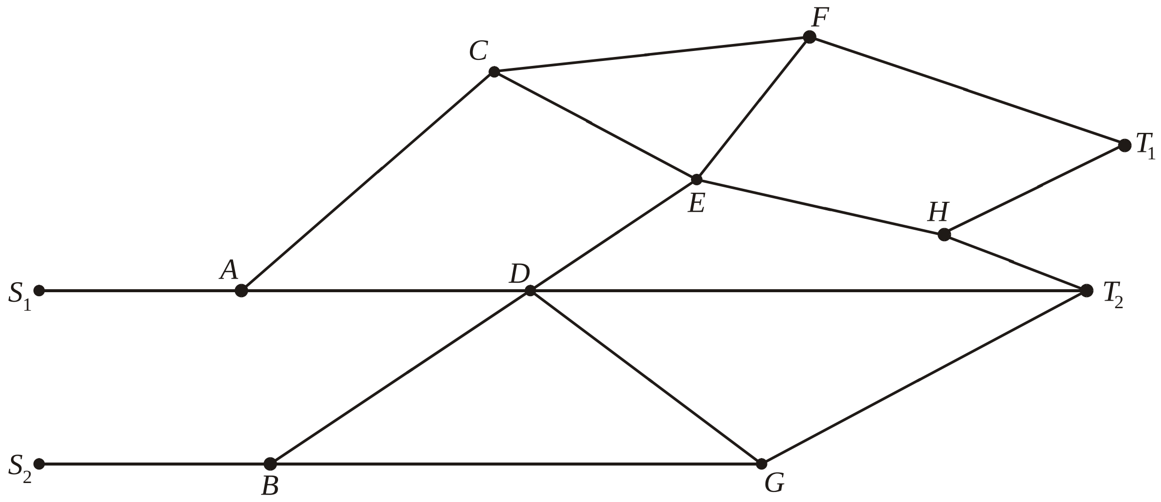
(c) Find the value of cuts C_1 and C_2.

(3)

Starting with the given feasible flow of 68,

(d) use the labelling procedure on Diagram 2 to find a maximal flow through this network. List each flow-augmenting route you use, together with its flow.

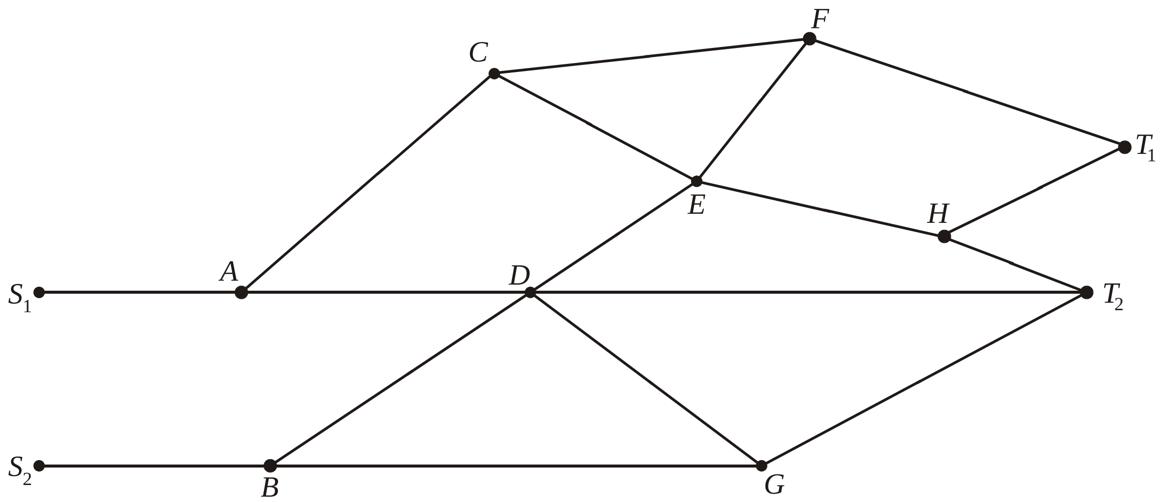
Diagram 2



(6)

(e) Show your maximal flow on Diagram 3 and state its value.

Diagram 3



(3)

(f) Prove that your flow is maximal.

(2)
(Total 18 marks)

17.

Figure 1

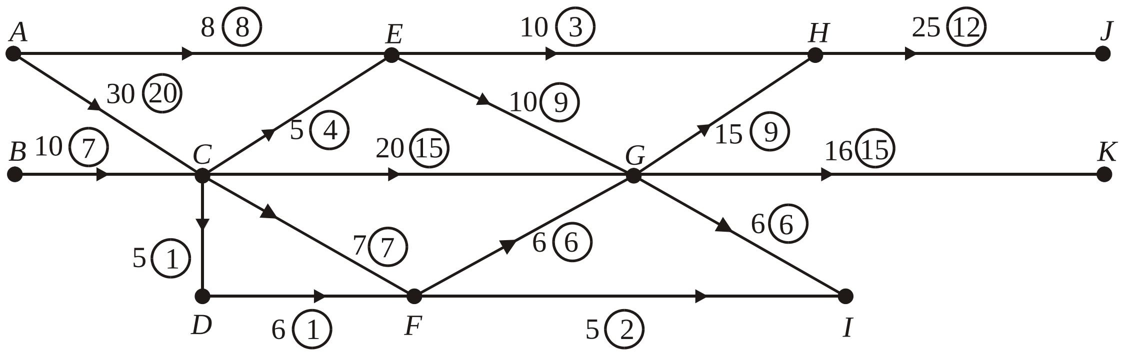


Figure 1 shows a capacitated directed network. The number on each arc is its capacity. The numbers in circles show a feasible flow from sources *A* and *B* to sinks *I*, *J* and *K*.

<u>**Take this as the initial flow pattern.**</u>

(a) On Diagram 1 below, add a supersource *S* and a supersink *W* to obtain a capacitated network with a single source and single sink. State the minimum capacities of the arcs you have added.

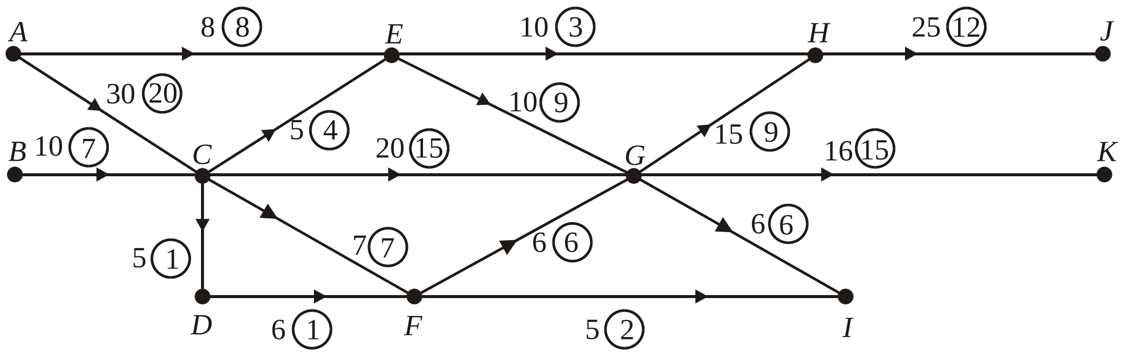


Diagram 1

(3)

(b) (i) Use the given initial flow and the labelling procedure on Diagram 2 to find the maximum flow through the network. You must list each flow-augmenting route you use together with its flow.

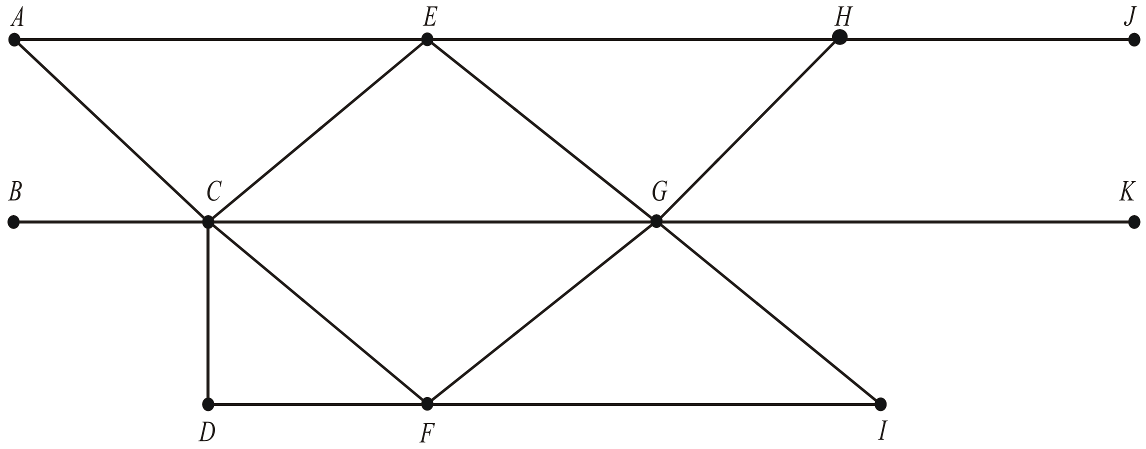


Diagram 2

(ii) Verify that your flow is maximal.

(9)

(c) Show your maximum flow pattern on Diagram 3.

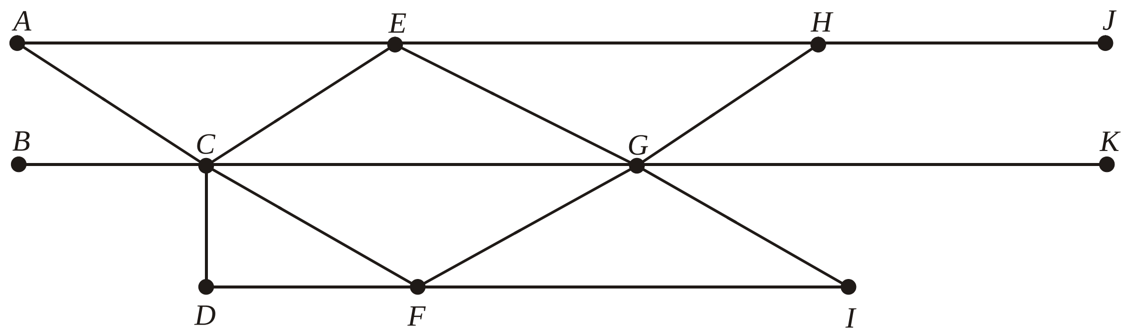


Diagram 3

(2)
(Total 14 marks)

18.

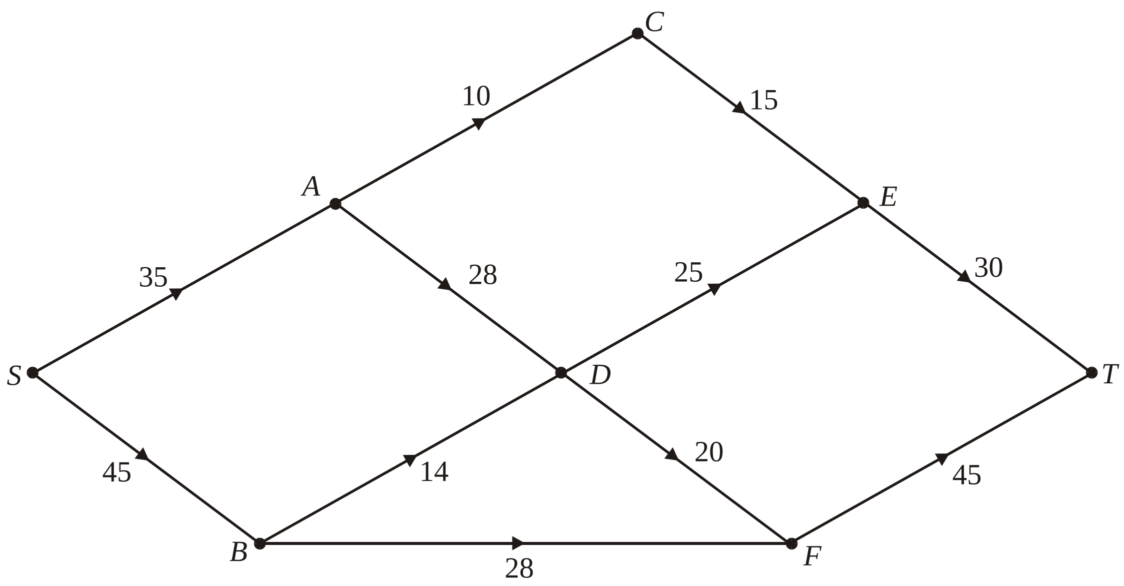


Fig. 1

Figure 1 shows a capacitated network. The number on each arc indicates the capacity of the arc.

(a) State the maximum flow along *SADET*

....

....

(1)

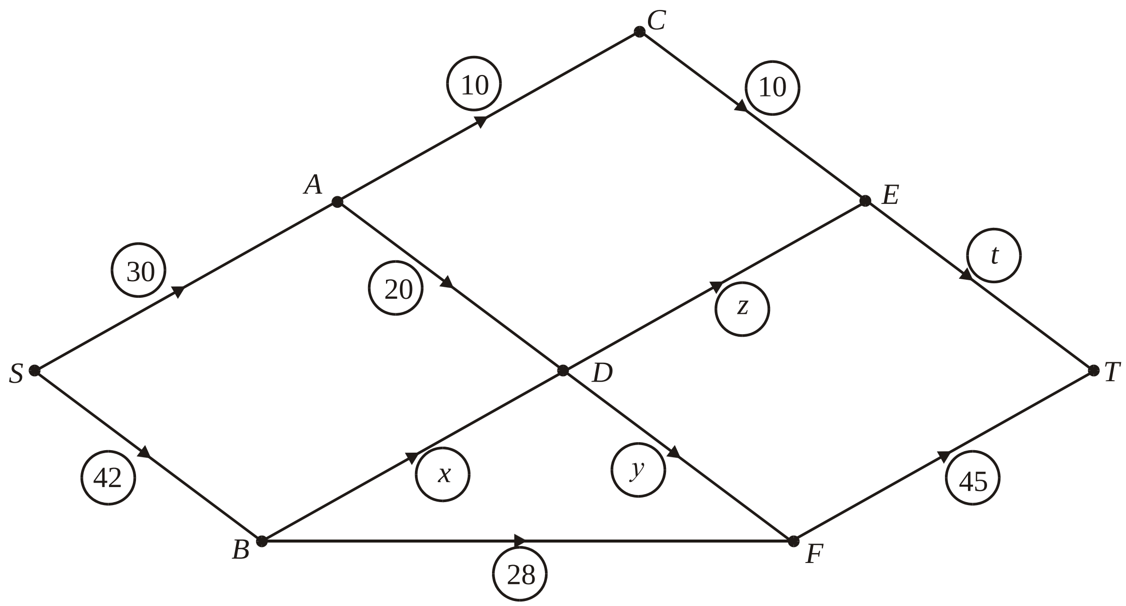


Fig. 2

Figure 2 shows a feasible flow of value 72 through the same network.

(b) Explaining your reasoning carefully, find the value of the flows x, y z and t.

....

....

....

....

....

(5)

(c) Explain why 72 is not the maximum flow

………………………………………………………………………………………….

………………………………………………………………………………………….

………………………………………………………………………………………….

………………………………………………………………………………………….

………………………………………………………………………………………….

(2)
(Total 8 marks)

1.	(a)	(By conservation of flow at *B*, *C* and *D*)	B3, 2ft 1ft 0	3
		$x = 11$ $y = 5$ $z = 12$		
		($\sqrt{x} - 6$) ($\sqrt{y} + 7$)		
	(b)	Flow is 31	B1	
		(max flow = min cut), cut through *AB*, *AC* and *SD*	B1	2

[5]

2.	(a)	Initial flow = 41	B1	1

Note

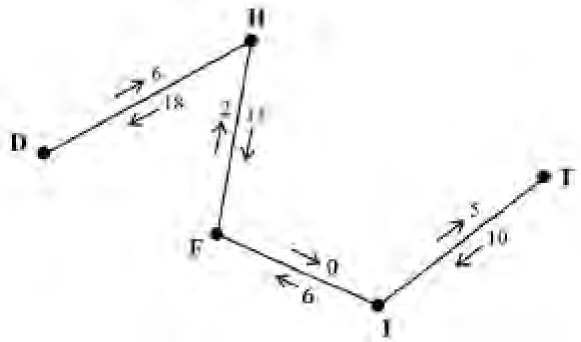
1B1: cao

(b)	Capacity of $C_1 = 69$	B1	
	Capacity of $C_2 = 64$	B1	2

Note

1B1: cao (permit B1 if 2 correct answers, but transposed)
2B1: cao

(c)



M1
A1 2

Note

1M1: Two numbers on each arc
1A1: cao

(d)	e.g. SBADHT – 2	M1 A1	
	SCGEDHT – 2	A1	3

Note

1M1: One valid flow augmenting route, S to T, found and value (≤ 4) stated.
1A1: Flow increased by at least 2
2A1: Flow increased by 4

(e)	maximum flow = minimum cut	DM1	
	e.g. cut through SA, SB, CE, GE, GI or HT, FI, GI	A1	2

Note

1DM1: Must have attempted (d) and made an attempt at a cut.
1A1: cut correct – may be drawn. Refer to max flow-min cut theorem three words out of four.

[10]

3.	(a)	Value of cut C_1 = 34; Value of cut C_2 = 45	B1; B1	2

Note

1B1: cao

2B1: cao

(b)	S B F G T or S B F E T – value 2	M1 A1	
	Maximum flow = 28	A1=B1	3

Note

1M1: feasible flow-augmenting route and a value stated

1A1: a correct flow-augmenting route and value

1A1= B1: cao

[5]

4.	(a)	$x = 9$, $y = 11$	B1, B1	2

1B1: cao (permit B1 if 2 correct answers, but transposed)

2B1: cao

(b)	AC DC DT ET	B2,1,0	2

1B1: correct (condone one error – omission or extra)

2B1: all correct (no omissions or extras)

(c)	36	B1	1

1B1: cao

(d)	$C_1 = 49$, $C_2 = 48$, $C_3 = 39$	B1, B1, B1	3

1B1: cao

2B1: cao

3B1: cao

(e)	e.g. SAECT	B1	1

1B1: A correct route (flow value of 1 given)

(f) maximum flow = minimum cut
cut through DT, DC, AC and AE — M1A1 — 2

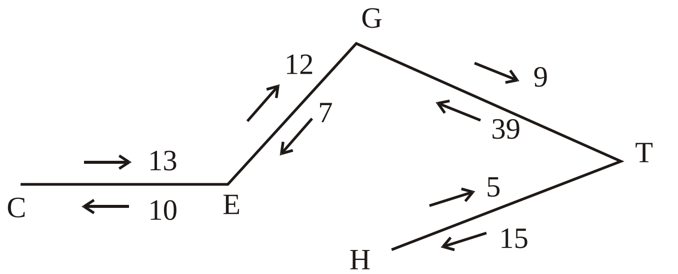
1M1: Must have attempted (e) and made an attempt at a cut.

1A1: cut correct – may be drawn. Refer to max flow-min cut theorem
three words out of fours.

[11]

5. (a) A cut divides the vertices into two sets,
one set containing the source(s) and the other the sinks(s) — B2,1,0 — 2

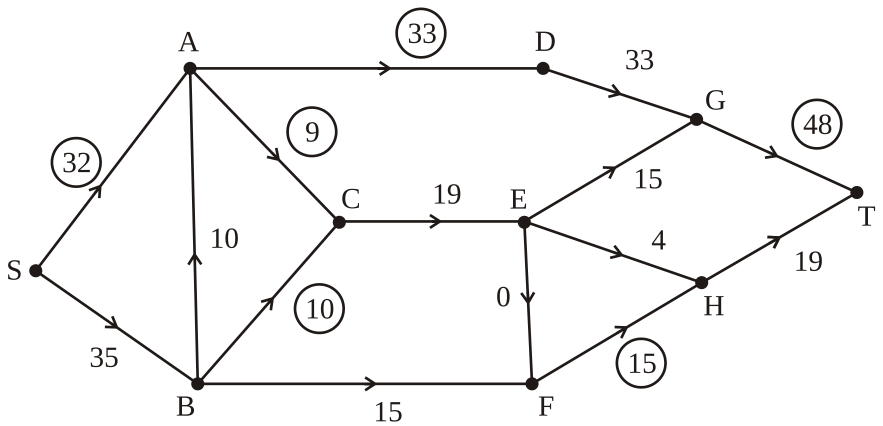
(b)



M1A1 — 2

(c) e.g. S B A C E G T – 9 — M1A1
S B A D G E H T – 1 — A1
S B F E H T – 3 — A1 — 4

(d)



M1A1 — 2

(e) Flow value 67 — B1 — 1

(f) Max flow – min cut theorem — M1
cut through AD, AC, BC, EF, FH — A1 — 2
or GT EH FH

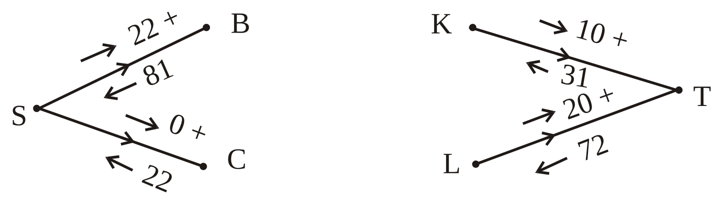
or SA BA BC EF FH

[13]

6.	(a)	85	B1	1
	(b)	$c_1 = 140$, $c_2 = 104$	B1, B1	2
	(c)	e.g.		
		S B D F H J T – 4	M1A1	
		S B D F G T – 1		
		S B D F C H I T – 2	A1	
		S B D F C H J T – 2	A1	
		S B D E G T – 10	A1	5
	(d)	Max flow – min cut theorem, flow is 104, min cut is c_2	M1A1	2

[10]

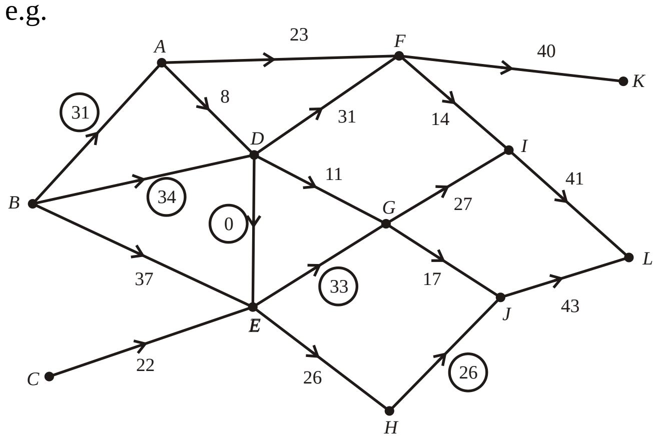
7. (a)



M1A1
A1 3

	(b)	<u>103</u>	B1	1
	(c)	e.g. S B E G I L T – 3	M1	
		S B E D F K T – 5	A4,3,2,1,0	5
		S B E H J G D F K T – 4		
		S B E G D F I L T – 9		

(d) e.g.

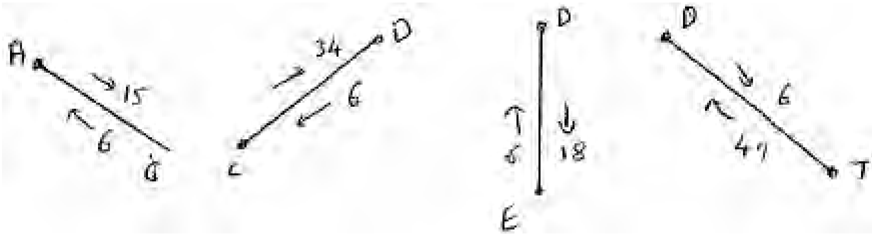


M1A1

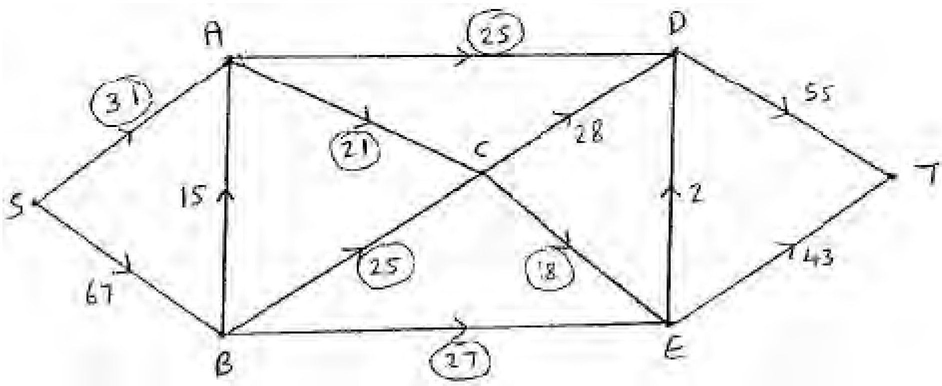
		Flow value <u>124</u> (given)	A1	3
	(e)	Max flow = min cut cut through <u>AB, BD, DE, EG, HJ</u>	M1A1	2

[14]

8.	(a)	$C_1 = 103$, $C_2 = 177$, flow = 76	B1, B1, B1	3
	(b)		M1A1	2



	(c)	e.g. S B C D T – 6	M1	
		S B C D E T – 1	A3,2,1,0	
		S B A C D E T – 15		
		Max flow is 98	B1	5
	(d)		M1A1	2



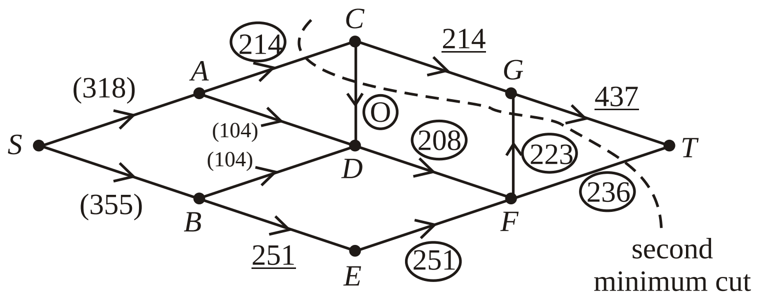
	(e)	Maximum flow = minimum cut	M1	
		Cut through AD, AC, BC and BE	A1	2

[14]

9.	(a)	(i) A cut is a division of the vertices of a flow network into 2 sets, one containing the source(s) and the other containing the sink(s).	B1	
		(ii) A cut whose capacity is least	B1	2

	(b)	$C_1 = 1038$, $C_2 = 673$	B1, B2, 0	3

(c) e.g.



M1A1A1 3

O = saturated

– = compulsory

	(d)	AC, CD, GF, FT	B1	1
	(e)	DE would not allow any further flow into EF DG would cross both minimum cuts – D can take extra flow, G can accept it. Flow increased by <u>8.6</u> to <u>759</u> (accept either number)	B1, 1, 0	2

[11]

10. (a) $SS_1 – 47$, $SS_2 – 87$, $T_1T – S_1$, $T_2T – T_3$ added to diagram 1 — M1 A1 — 2

If all 4 nos. zero then M0

M1 4 arcs added correctly + 4 numbers given (diagram 1 only) condone lack of arrows

A1 c.a.o. (diagram 1 only) penalise arrow errors here

(b)

SS_1 $\rightarrow 0$, SS_2 $\rightarrow 38$, T_1T $\rightarrow 8$, T_2T $\rightarrow 20$ — M1 A1 — 2

$\leftarrow 47$ $\leftarrow 49$ $\leftarrow 43$ $\leftarrow 53$

M1 4 arcs, 2 numbers and 2 arrows $\leftrightarrows$ *per arc*

A1 c.a.o.

(c) e.g. $S\ S_2\ A\ D\ T_1\ T$ — 2
$S\ S_2\ C\ E\ T_2\ T$ — 1 M1
$S\ S_2\ C\ E\ D\ T_2\ T$ — 10 A4,3,2,1,0
$S\ S_2\ C\ E\ B\ D\ T_1\ T$ — 4
Maximum flow — 113 B1 6

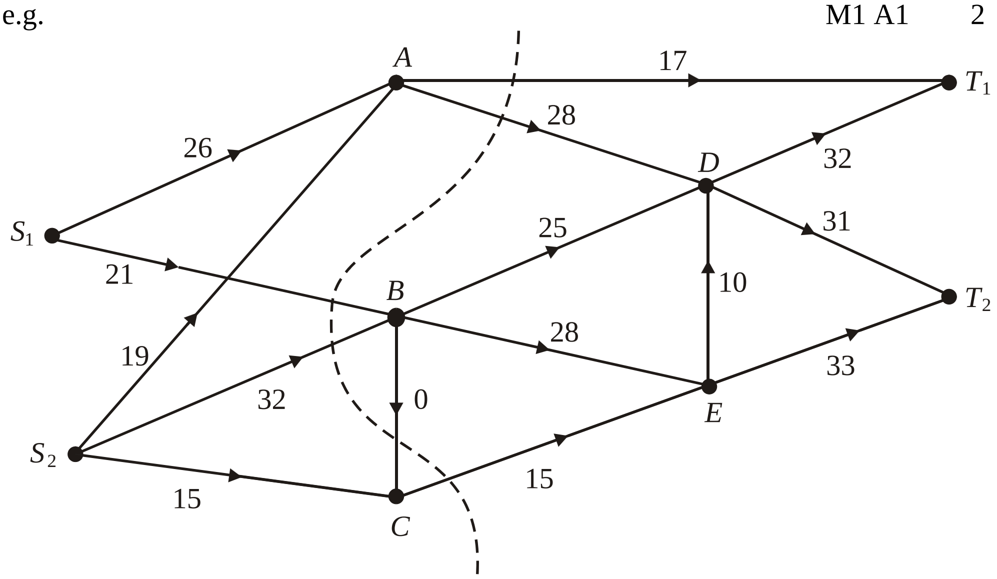
M1 2 correct routes + flows found (flow > 10 gets M0) (condone initial f.a. routes <u>only</u> if clearly repeated from new ones)

A4 all flows + routes to 15 more <u>or</u> flow increased above 17 more

A2 ≥ 3 flows + routes to 11 more <u>or</u>
A1 at least 2 flows + routes found to 5 more

B1 113 c.a.o.

(d) e.g. M1 A1 2



M1 consistent flow of 101(*), complete clear (doesn't need to ft from (c))

A1 correct flow of 113 including arrows

(e) Max flow – min cut theorem; cut AT_1, AD, S_1B, S_2B, BC, CE M1 A1 2

M1 flow of 113 + cut attempted + max flow – min cut theorem referred to (3 out of 4)

A1 c.a.o.

(f) Idea of a <u>directed</u> flow along arcs; from S to T; through a system/network; practical B2,1,0 2

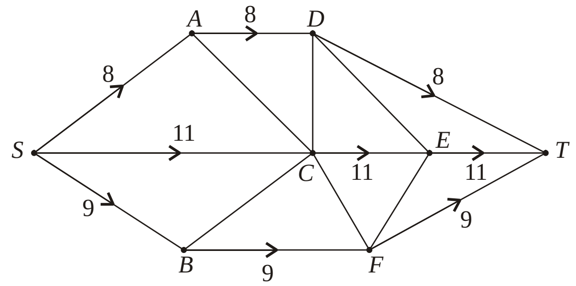
B2 all 4 bits there

B1 2 out of 4 there

[16]

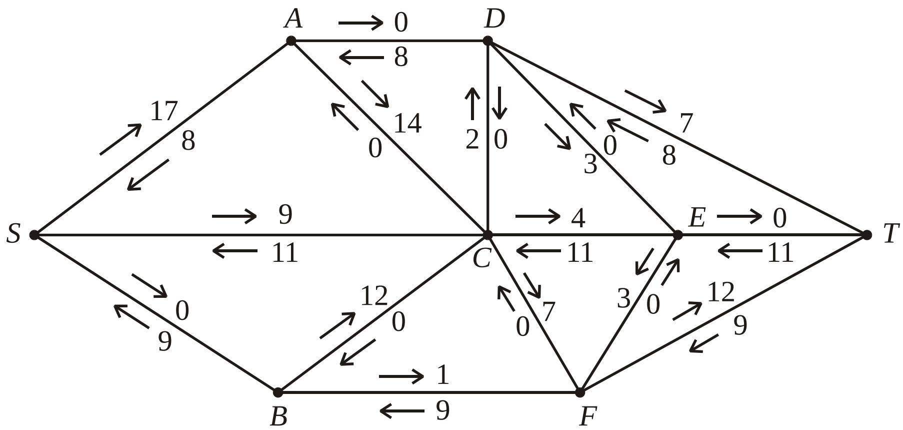
11. (a) SADT – 8 SCET – 11 SBFT – 9 B2, 1, 0

(b)



B1 3

(c) (i)

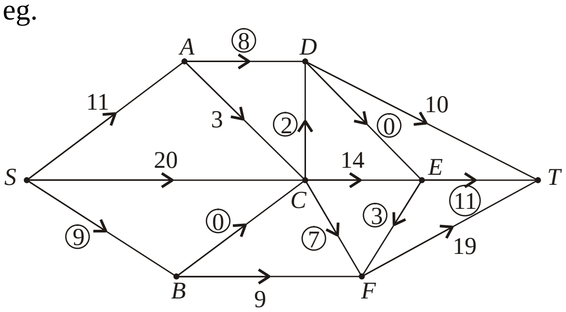


M1
A1 2

e.g.
S A C D T – 2 S C F T – 6 A1
S A C E F T – 3 S A C F T – 1 A1 3

max flow 40

(ii) eg.



M1 A1 2

	(iii)	Max flow – min cut theorem	M1	
		cut AD, CD, DE, ET, CF, BC, SB ie {S A C E } {B D F T}	A2, 0	3
	(d)	Idea of a <u>directed</u> flow through a <u>system</u> of arcs from <u>S to T</u> <u>practical</u>	B1	1

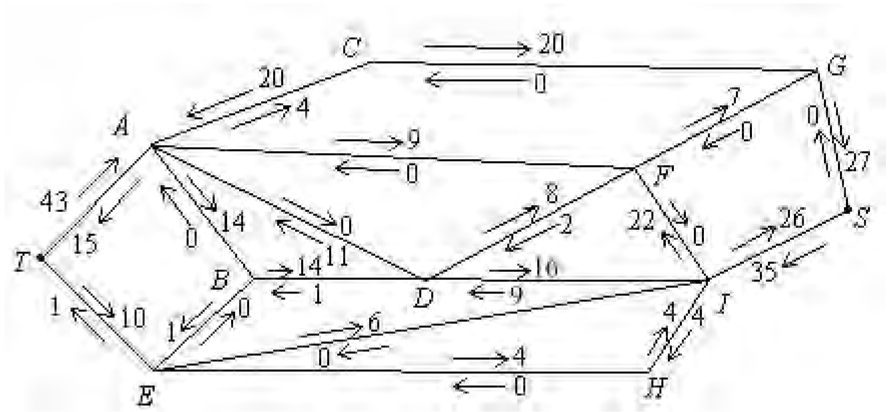
[14]

12.	(a)	(By conservation of flow at *B*, *C* and *D*)	B3,2ft,1ft,0	3
		<u>x = 1</u> <u>y = 5</u> <u>z = 12</u>		
		($\sqrt{}x$ – 6) ($\sqrt{}y$ + 7)		
	(b)	<u>Flow is 31</u>	B1	
		(max flow = min cut), cut through <u>*AB*, *AC* and *SD*</u>	B1	2

[5]

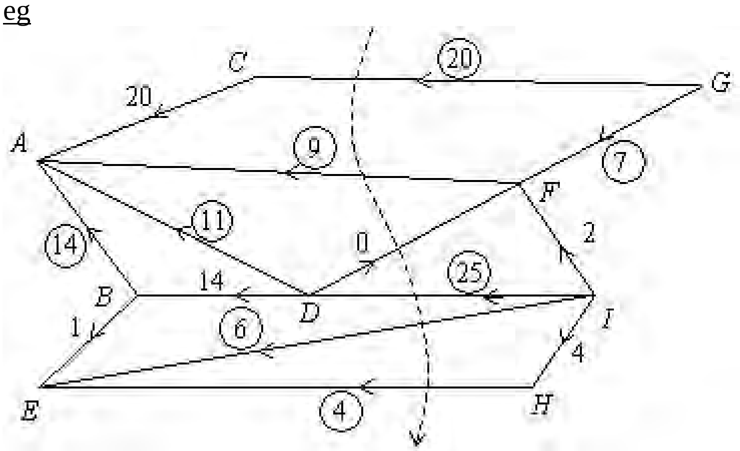
13.	(a)	x = 9, y = 16	B1 B1	2
	(b)	Initial flow = 53 – Either finds a flow-augmenting route or demonstrates not enough saturated arcs for a minimum cut	B1 B1	2

(c)



	M1A1	2
e.g. *IDA* – 9	A1	
IFDA – 2	A1	
max flow – 64	B1	3

	(d)	eg	M1 A1	2

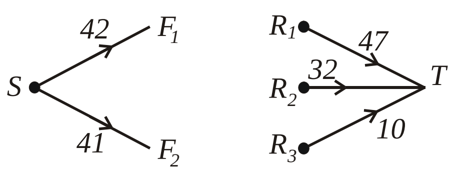


	(e)	Max flow – min cut	M1	
		Finds a cut *GC*, *AF*, *DF*, *DJ*, *EI*, *EH* value 64	A1	2
		Note: must not use supersource or supersink arcs.		

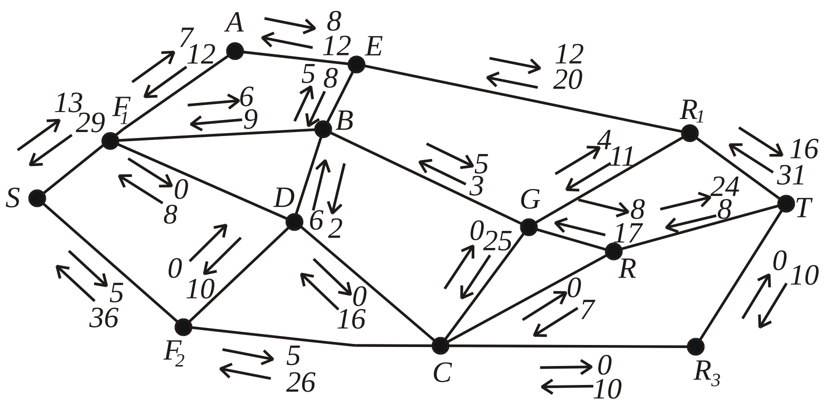
[13]

14.	(a)	$C_1 = 7 + 14 + 0 + 14 = 35$	B1	
		$C_2 = 7 + 14 + 5 = 26$	B1	
		$C_3 = 8 + 9 + 6 +8 = 31$	B1	3
	(b)	Either Min cut = Max flow and we have a flow of 26 and a cut of 26 or C2 is through saturated arcs	B1	1
	(c)	Using EJ (capacity 5) e. g – will increase flow by 1– ie increase it to 27 since only one more unit can leave E. - BEJL – 1	M1 A1	
		Using FH (capacity 3) e. g.– will increase flow by 2 – ie increase it to 28 since only two more units can leave F. - BFHJL - 2		
		Thus choose option 2 add FH capacity 3.	A1	3

[7]

15.	(a)	$x = 3$, $y = 26$	B1, B1	2
	(b)		M1 A1 A1	3

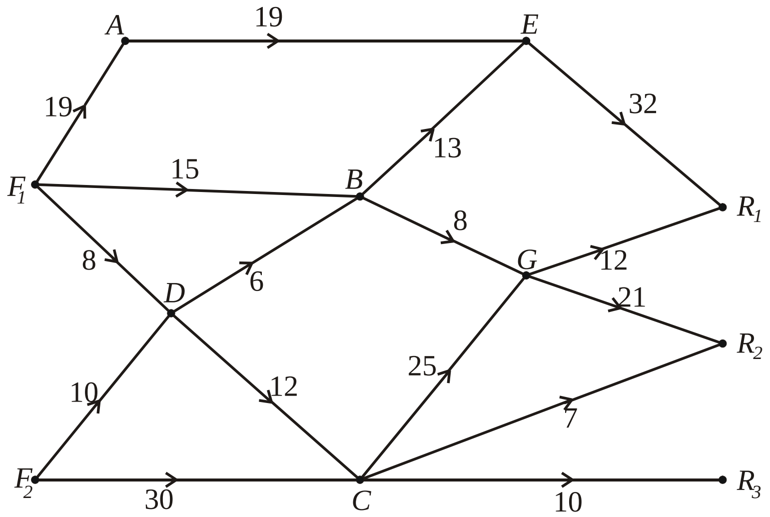
(c)



M1 A1 ft A1 ft 3

<u>e.g.</u>	S F_1 A E R_1 T – 7	DM1	
	S F_1 B E R_1 T – 5	A1	
	S F_1 B G R_1 T – 1	A1	
	S F_2 C D B G R_2 T – 4	A1	4

(d) <u>e.g.</u>



B1

<u>Max Flow 82</u> B1 2

(e) <u>e.g.</u>

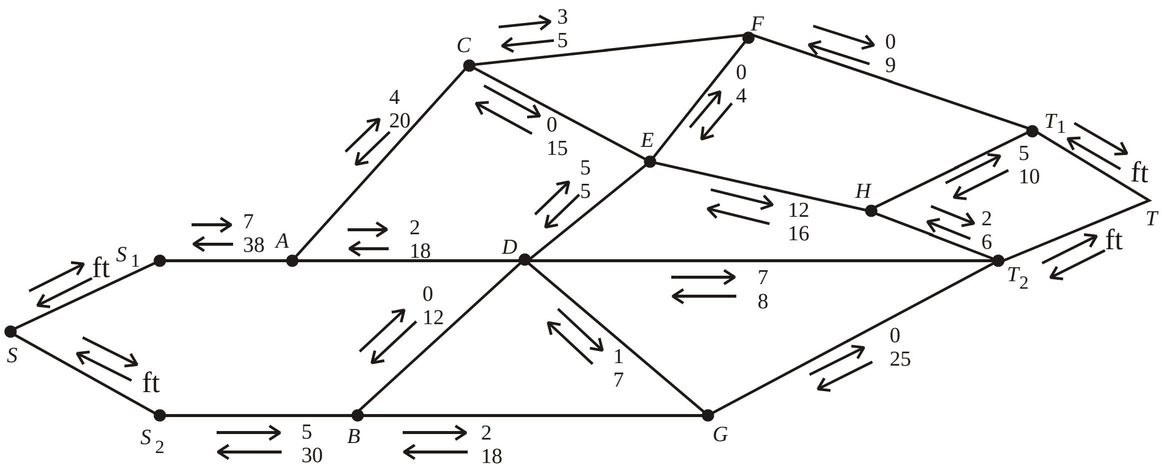
F_1 A, BE, BG, CG, CR_2, CR_3 (=82)

Or ER_1, BG, CG, CR_2, CR_3 (=82) M1 A1 2

[16]

16.	(a)	Adds S and T and arcs	M1	
		$SS_1 \geq 45$, $SS_2 \geq 35$, $T_1T \geq 24$, $T_2T \geq 58$	A1	2
	(b)	Using conservation of flow through vertices $x = 16$ and $y = 7$	B1 B1	2
	(c)	$C_1 = 86$, $C_2 = 81$	B1 B2	3

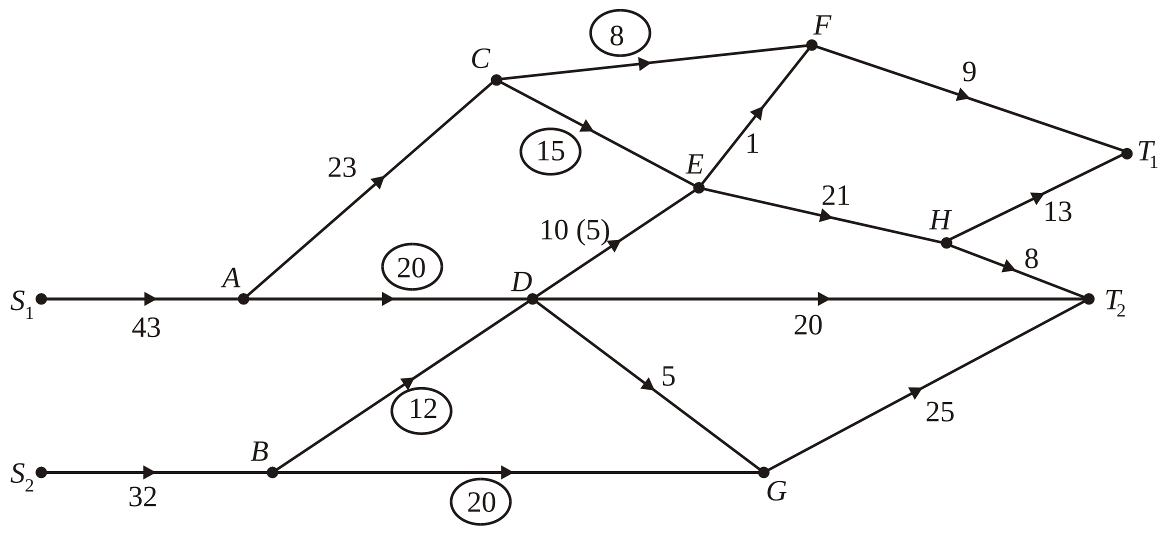
(d)



M1 A1
dM1

e.g.	$S\,S_1\,A\,D\,E\,H\,T_2\,T$	-2	A1	
	$S\,S_1\,A\,C\,F\,E\,H\,T_1\,T$	-3	A1	
	$S\,S_2\,B\,G\,D\,T_2\,T$	-2	A1	6

(e) e.g.:

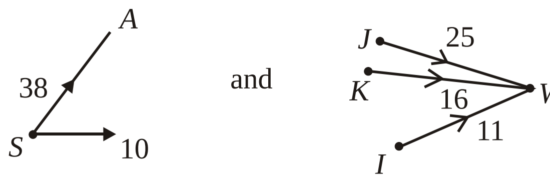


M1 A1

<u>Flow 75</u> A1 3

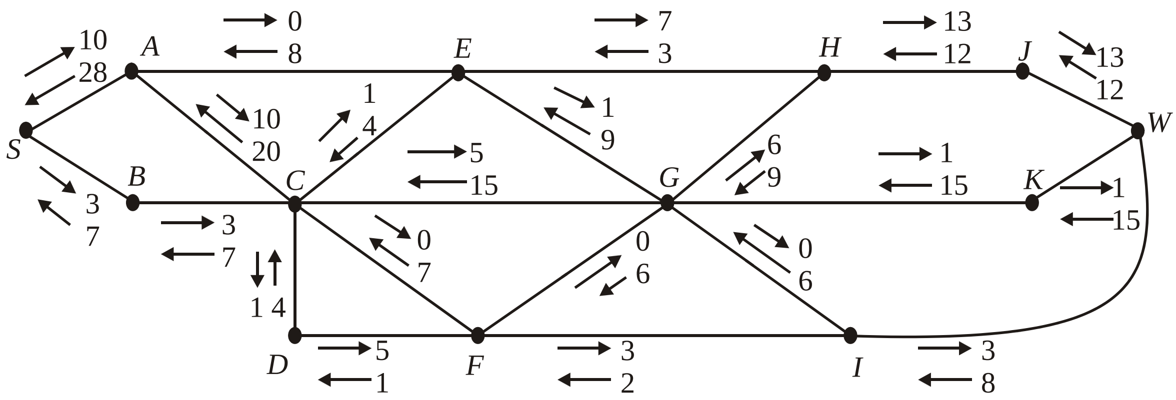
(f)	Max flow – min cut theorem cut through		dM1	
	CF, *CE*, *AD*, *BD*, *BG* (value 75)		A1	2
				[18]

17. (a)



M1 A1 A1 3

(b) (i) M1 A1 A1 3

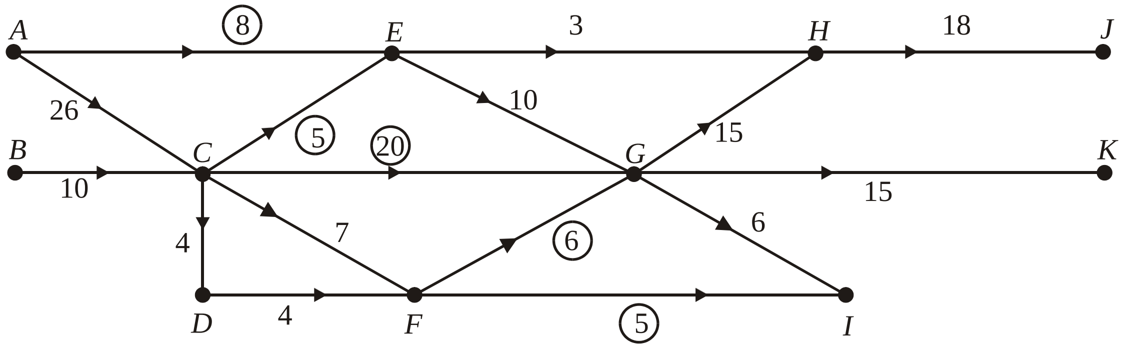


For example,

S B C D F I W – 3	M1 A1	
S A C G H J W – 5	A1	
S A C E G H J W – 1	A1	4

(ii)	Maximum flow 44	B1	
	States valid cut *AE*, *CE*, *CG*, *FG*, *FI*	B1	2

(c)



M1 A1 2

[14]

18. (a) Max flow along *SADET*

= min (35, 28, 25, 30) = 25 B1 1

(b) Using flow conservation at vertices,

At B $42 + 28 + x$, $x = 14$

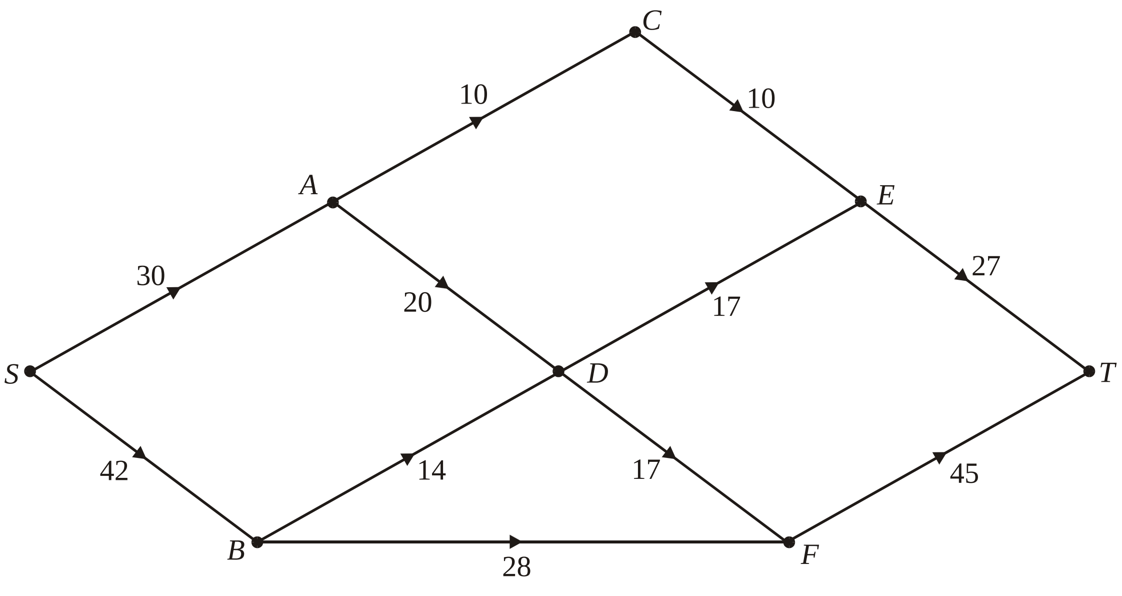
At F $28 + y$, $y = 17$ M1 A1 A1

At D $20 + x = y + z$

$20 + 14 = 17 + z$, $z = 17$ A1

At E $10 + z = t$, $t = 27$ A1 5

So pattern is



(c) 72 is not the maximum flow since it is possible to find an additional flow.

On *SADET* we have spare capacities *SA* (35—30), *AD* (28—20), *DE* (25—17), *ET* (30—27), i.e. *SA* (5), *AD* (8), *DE* (8), *ET* (3).

So a further flow of 3 is possible along this route. M1 A1 2

[8]

1. No Report available for this question.

2. Again this question was a good source of marks for many candidates. Most found the flow of 41, but the correct cut capacities of 69 and 64, were not seen often.

A few omitted one of the pairs of labels in (c). Most were able to find one correct flow augmenting route and many of these went on to find a second route, but many incorrect routes were seen.

Specific reference to the Max flow-min cut theorem was required in part (e) as well as finding a minimum cut.

3. Most candidates were able to evaluate at least one cut correctly, but only the best were able to calculate both accurately. Many candidates were able to gain full marks in part (b), with many simply listing the flow-augmenting route, its flow and the maximum flow. A significant number complicated the question by not starting from the given flow, provided in (a).

4. Candidates often answered parts (a) (c) and (e) correctly. Part (b) was well answered by most but some omitted at least one arc. Part (d) proved the most challenging, although good answers were seen many used the flow numbers rather than the capacity numbers. In part (f) candidates needed to state a minimum cut and refer to the max flow-min cut theorem, few did both.

5. This question proved challenging for many candidates. In part (a) candidates often answered a different question and described the method used to calculate the value of a cut, of those who answered the question set many were able to gain some credit but few gained full credit for a detailed answer. Most were able to answer part (b) correctly and then able to find at least one correct flow-augmenting route and its value in part (c). Few were able to find all routes and flows up to 13 extra, although most coped with the routes involving backflow much more competently, some attempted to reverse or redirect the flow along individual arcs or send flows of negative value though the system.

The direction of flow along EF was often misinterpreted, or its value omitted, in part (d), but most candidates were able to produce a consistent diagram. Double arrows and numbers should not be used on a diagram showing the final flow pattern. In part (f) candidates are expected to refer to the max flow-min cut theorem and this part is therefore only available to those candidates who have found the maximum flow. There were three minimum cuts and most candidates were able to state one of them, generally those who drew the position of a minimum cut on their diagram in part (d) gained the marks more efficiently. Some candidates who attempted to list the arcs in a minimum cut omitted one of the arcs.

6. Although many good answers to parts (a) and (b) were seen, disappointingly many candidates were not able to find the value of the initial flow or evaluate the cuts correctly. Most candidates were able to find at least one flow–augmenting path, but few found all paths to 19, the ‘backflow’ paths causing most of the problems. Some assumed that I was the sink, rather than T. Very few candidates gained credit in (d), many had the right idea but either didn’t state a cut, or didn't quote the theorem.

7. The great majority of the candidates were able to secure the first four marks, for parts (a) and (b). The arcs were usually labelled correctly with 2 values. Where these were incorrect, zeros often appeared. Candidates frequently obliterated or modified existing values, thus rendering them difficult or impossible to discern. The initial flow was correctly identified by most as 103, and the flow-augmenting route SBEGILT with flow 3 was a popular choice, at which point the question finished for a great number of candidates. All the remaining flow-augmenting routes involved backflows and many candidates were unable to make progress, even though it was stated in the question that they should be seeking to increase the flow to 124. Those who did understand the use of backflows were usually able to find correct flow-augmenting routes, with the odd arithmetic slips seen. Few candidates, even those who found correct routes to 124 were able to draw a correct flow in part (d), the commonest slip was to have a flow of 5 from D to E. Only a very few were able to link the maximum flow with a correct minimum cut.

8. A surprisingly few candidates gained all three marks in part (a), with 177 rarely seen and 103 seen only slightly more frequently. Most candidates completed the labelling correctly; the only commonly seen error was that of swapping the labels on DE. Most candidates were able to find one alternating path but only the better candidates found all, many over-saturated DT. Many who attempted diagram 2 in part (d) didn't check their flows into and out of nodes. Candidates had to be looking at a flow of 98 to gain any credit in part (e), although some clearly felt they managed to prove their flow of 82 was maximal. A disappointing number of good candidates found the correct maximum flow and correct minimum cut but did not use the theorem to link them together.

9. This was often quite poorly done. Extremely poor use of technical terms and some very confused explanations were seen in part (a). Most, but by no means all, were able to state the value of C1 correctly but C2 proved more difficult. Many ignored the direction of the flow and simply found the sum of all the arcs cut, including that of CD which does not flow into the cut, and therefore should not be included, giving the very common incorrect answer of 802 for cut C2. Many did not make the link between parts (b) and (c) and wasted time using the labelling procedure and seeking flow-augmenting routes, rather than using the minimum cut to deduce the maximum flow pattern. This often led to time problems for these candidates later n in the paper. Few candidates gave a clear, unambiguous, diagram showing the maximum flow. Only a few found the correct second minimum cut, and many used the flows rather than the capacities or each arc. In part (e) only a few adequately explained that arc EF was capacitated (or both FT and FG were capacitated.) Few gave the correct value of the increased flow.

10. Most candidates dealt with the supersource and supersink correctly and added the correct numbers on the edges, but many omitted the arrows in part (a) and made arithmetic slips in part (b). Most candidates were able to increase the flow to 109 correctly, but a large number of candidates tried to use routes involving a flow from C to B. Candidates should not list the routes that give the initial flow as part of their new augmented flow route list, but should give the value of the flow they send along each route they use. Part (d) was often poorly done with many sending flow from C to B as the commonest error. Of those who found the correct maximum flow only a very few obtained the correct minimum cut and linked the two with the max-flow min-cut theorem. Most candidates were able to make some progress with part (f) but very few gave complete answers.

11. Part (a) was well answered by the vast majority of the candidates, with only a very few finding the sum of the values along the route. In part (b) the diagrams were often unnecessarily complicated by capacities, double arrows and arrows in the reverse direction. The initial labelling in part (c) was usually well done, although some omitted arcs BC and DE. As always candidates should avoid obliterating the initial values, the examiners have to try to read this to give credit! Some did not find all the flow augmenting routes, some found the routes but did not state the flow, and others did not update their diagrams and so oversaturated some arcs. Many failed to put arrows onto their diagram in (ii), or omitted the flow along one arc (often CE). Only the very best candidates were able to prove that their flow was maximal. Part (d) was usually poorly answered.

12. This was well-answered in general. Relatively few candidates found a correct cut, some didn't state any cut, but many used the value of the flow rather than the capacity.

13. This was frequently poorly done. Part (a) was usually correct but 7 and 14 were often seen. The flow in part (b) was often correct, but candidates were generally not able to give an explanation. Part (c) was poorly done. In the diagram incorrect or transposed numbers were given in the initialisation of the labelling procedure, especially on arc DF. Many attempts were made to find flow augmenting routes where an arc was already saturated or had been saturated by an earlier route. Many candidates stated incorrect maximum flows with 62 and 65 popular wrong answers. Many candidates did not produce an internally consistent flow diagram with the flow through F a particular source of inconsistency, and many omitted arrows. Few candidates gained full credit here, some candidates tried to find a cut using the values given in their final flow diagram – rather than on the initial diagram, some found the correct minimum cut – passing through saturated arcs, but omitted arc DF.

14. Most candidates were able to state the correct value of C2 but errors in C1 and C3 were common, with values of 40, 42 and 33 being popular for the former and 26 and 25 popular for the latter. Part (b) was usually well answered. Part (c) was well answered by the majority of the candidates with most considering both options and stating the flow-augmenting paths each arc made possible. Arguments using cuts were rarely successful. Once again some candidates did not state their conclusion.

15. Most candidates gained full marks for part (a). In part (b) some candidates omitted arrows and some stated the value of the flow on the new arcs rather than their capacity. It was very difficult, and sometimes impossible, to read off the starting numbers on some diagrams for part (c) the examiners need to be able to read these if marks are to be awarded. Many candidates were able to increase the flow by 13, but only the more able were able to increase the flow by 17. A number of candidates did not start from the given flow and lost marks accordingly. Most of the diagrams in part (d) showed a flow of 78 rather than 82. In part (e) some candidates used flows rather than capacities to work out the value of cuts – only those who found a flow of 82 were able to find a suitable cut. Some candidates tried to use cuts through the supersource/ supersink arcs they added to the diagram in part (b); this is not permitted.

16. Once again this question was either very well or very poorly attempted. Many candidates ran out of time and did not complete the question, but it was a rich source of marks for some candidates. In part (a) most candidates correctly drew in the arcs from the supersource and supersink but a minority omitted the weights, or put incorrect weights on them, usually basing their numbers on the given initial flow, rather than on the capacity of the arcs. Most candidates were able to score full marks in part (b). In part (c) few correct answers were seen, with cut 2 being the rarest correct answer, a lot of candidates used the initial flow rather than the capacity of each arc. Of those who correctly used the labelling procedure in part (d) the commonest error was in not spotting the back-flow routes and hence only increasing the flow by 2, very few managed to increase the flow to 75. The most common error in part (e) was to omit the value along one arc – often *EF*, leading to an inconsistent flow pattern. Unless candidates had found the correct maximum flow it was impossible to use the minimum cut – maximum flow theorem to prove that their flow was maximal in part (f). Many candidates quoted the theorem but gained no credit unless they applied it.

17. Some very good solutions to this question were seen and also some very poor solutions. Many candidates found part (a) difficult, with incorrect values stated on the arcs, many linking *S* to *D* and many omitting arrows. Success in part (b) depended upon use of the labelling procedure. Candidates who used the labelling procedure correctly usually scored well, although some errors in calculating the initial values of the labels were seen. The examiners must be able to read these initial flow labelling values if full credit is to be gained, some candidates delete these initial values so enthusiastically they become illegible. Some candidates did not use the given initial flow as the start for their flow-augmenting routes, often listing every route they could think of, and others forgot to include *S* and *W* in their routes. Those who found the correct maximum flow often stated a correct minimum cut. In part (c) many candidates omitted arrows, some omitted the flow along at least one arc, and others gave an inconsistent flow pattern (often revealed by the flow into and out of *C* not being equal).

18. No Report available for this question.